

# Industrious Number Theory

March 11 – 13, 2009

at

Seminar Room 1423  
Korea Institute for Advanced Study

Organized by

CHANG Seunghwan (Yonsei University)

CHOI Youn-Seo (KIAS)

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## Time Table

	3月11日(水)	3月12日(木)	3月13日(金)
9:30–10:30	Asakura	Jang	Harada
10:50–11:50	Im	Yamazaki	Lee
2:00–3:00	Han	Yoshida	Sun
3:20–4:20	Nakamura	Hattori	Otsuki
4:40–5:40	Moon	Choi	Hiranouchi

## Program

### 3月11日(水)

- 9:30–10:30 ASAKURA Masanori (Hokkaido Univ.),  
*An elliptic K3 over  $\mathbb{Q}_p$  with finitely many torsion 0-cycles*
- 10:50–11:50 IM Bo-Hae (Chung-Ang Univ.)  
*Weak approximation for linear systems of quadrics*
- 11:50–14:00 Lunch
- 14:00–15:00 HAN Lin (Inha Univ.)  
*Examples of semistable abelian varieties over  $\mathbb{Q}$  with good reduction outside a single prime*
- 15:20–16:20 NAKAMURA Kentaro (Tokyo Univ.)  
*Classification of 2-dimensional split trianguline representations of  $p$ -adic fields*
- 16:40–17:40 MOON Hyunsuk (Kyungpook Univ.)  
*On the structure of the Mordell-Weil groups of Jacobians over infinite number fields*

### 3月12日(木)

- 9:30–10:30 JANG Junmyeong (KIAS)  
*Semi-stable fibrations of generic  $p$ -rank 0 and  $p$ -rank reduction problems*
- 10:50–11:50 YAMAZAKI Takao (Tohoku Univ.)  
*Counter examples to variants of the Milnor-Bloch-Kato conjecture*
- 11:50–14:00 Lunch
- 14:00–15:00 YOSHIDA Manabu (Kyushu Univ.)  
*Ramification of local fields and Fontaine's property ( $P_m$ )*
- 15:20–16:20 HATTORI Shin (Kyushu Univ.)  
*On a ramification bound of semi-stable torsion representations over a local field*
- 16:40–17:40 CHOI Dohoon (Aerospace Univ.)  
*Congruence for Siegel modular forms*

**3月13日(金)**

9:30–10:30 HARADA Shinya (Kyushu Univ.)

*Hasse-Weil zeta function of absolutely irreducible  $SL_2$ -representations of the figure 8 knot group*

10:50–11:50 LEE Dong Uk (KIAS)

*Nonemptiness of the  $\mu$ -ordinary locus of Shimura varieties of Hodge type*

11:50–14:00 Lunch

14:00–15:00 SUN Hae-Sang (KIAS)

*A cuspidal class number of the tower of modular curves*

15:20–16:20 OTSUKI Rei (Keio Univ.)

*A homomorphism concerning two systems for an elliptic curve*

16:40–17:40 HIRANOUCI Toshiro (RIMS, Kyoto Univ.)

*Flat modules and Gröbner bases over truncated discrete valuation rings*

## Abstracts

### ASAKURA Masanori

**Title:** An elliptic K3 over  $\mathbb{Q}_p$  with finitely many torsion 0-cycles

**Abstract:** This is a joint work with Kanetomo Sato. In this talk, I show that for the elliptic K3 surface  $X$  defined by an equation  $3y^2 + x^3 + (3x + 4t^4) = 0$  over  $\mathbb{Q}_p$  the  $p$ -adic regulator  $K_1(X) \otimes \mathbb{Q}_p \rightarrow H_g^1(\mathbb{Q}_p, H^2(X, \mathbb{Q}_p(2)))$  is surjective if  $p = 7, 11, 19, 23, 31$ . Here “ $H_g^1$ ” denotes the  $g$ -part of Bloch-Kato. As a consequence, the torsion subgroup of  $\text{CH}^0(X)$  is finite.

### CHOI Dohoon

**Title:** Congruence for Siegel modular forms

**Abstract:** We employ recent results on Jacobi forms to investigate congruences and filtrations of Siegel modular forms of degree 2. In particular, we determine when an analog of Atkin’s  $U(p)$ -operator applied to a Siegel modular form of degree 2 is nonzero modulo a prime  $p$ . Furthermore, we discuss explicit examples to illustrate our results. This is joint work with Y. Choie and O. Richter.

### HAN Lin

**Title:** Examples of semistable abelian varieties over  $\mathbb{Q}$  with good reduction outside a single prime

**Abstract:** In this talk, I will introduce results on examples of semistable abelian varieties over  $\mathbb{Q}$  using the arguments of Fontaine and Schoof. In order to specify an abelian variety  $A$  over  $\mathbb{Q}$  with good reduction outside a prime  $p$  up to isogeny, it’s enough to construct its  $\ell$ -divisible group. To do that, we need the following information:

1. Classification of the finite flat simple group schemes of  $\ell$ -power order over  $\mathbb{Z}[1/p]$  arising as a subgroup of  $\ell$ -torsion of abelian varieties;
2. Classification of a certain class of extensions of such simple group schemes.

These problems boil down to finding, via class field theory, all the Galois extensions over  $\mathbb{Q}$  unramified outside  $p$  and  $\ell$ , satisfying bounds of Fontaine and Odlyzko with additional properties.

### HARADA Shinya

**Title:** Hasse-Weil zeta function of absolutely irreducible  $\text{SL}_2$ -representations of the figure 8 knot group

**Abstract:** In this talk, we will give explicit descriptions of Weil-type zeta functions defined by the numbers of  $\text{GL}_2$ -conjugacy classes of absolutely irreducible  $\text{SL}_2$ -representations of the figure 8 knot group over

finite fields. Then we prove that the Hasse-Weil type zeta function of absolutely irreducible  $SL_2$ -representations of the figure 8 knot group over finite fields has interesting arithmetic properties such as meromorphic continuation and a functional equation, and its central value has relations with other knot invariants.

### HATTORI Shin

**Title:** On a ramification bound of semi-stable torsion representations over a local field

**Abstract:** Let  $k$  be a perfect field of characteristic  $p > 0$ ,  $K_0 = \text{Frac}(W(k))$  and  $K$  be a finite totally ramified extension of  $K_0$  of degree  $e$ . Let  $r$  be a non-negative integer with  $r < p - 1$ . In this paper, we give a bound of the upper ramification of the torsion representations of the semi-stable  $p$ -adic representations with Hodge-Tate weights in  $\{0, \dots, r\}$ . Namely, we show the upper numbering ramification group  $G(j)$  (à la Fontaine) acts trivially on the  $p^n$ -torsion representations for  $j > u(K, r, n)$ , where  $u(K, 0, n) = 0$ ,  $u(K, 1, n) = 1 + e(n + 1/(p - 1))$  and  $u(K, r, n) = 1 - p - n + e(n + r/(p - 1))$ .

### HIRANOUCHI Toshiro

**Title:** Flat modules and Gröbner bases over truncated discrete valuation rings (joint work with Y. Taguchi)

**Abstract:** We present basic properties of Gröbner bases of submodules of a free module of finite rank over a polynomial ring with coefficients in a tdvr ( $:=$  truncated discrete valuations ring). As an application, we give a criterion for an algebra of finite type over a tdvr to be flat and prove the existence of a flat lifting of a flat algebra over a tdvr.

### IM Bo-Hae

**Title:** Weak approximation for linear systems of quadrics

**Abstract:** We give local conditions at  $\infty$  ensuring that ] the intersection of  $n$  quadrics in  $\mathbb{P}^N$ ,  $N \geq n$ , satisfies weak approximation. This is the joint work with Michael Larsen (at Indiana University).

### JANG Junmyeong

**Title:** Semi-stable fibrations of generic  $p$ -rank 0 and  $p$ -rank reduction problems

**Abstract:** For a semi-stable fibration from a proper smooth surface to a proper smooth curve over the field of complex numbers, the semi-positivity theorem holds. But over a field of positive characteristic, the semi-positivity theorem is not valid in general. In this talk, we will see for a semi-stable fibration of generic  $p$ -rank 0, the semi-positivity theorem fails after sufficiently many Frobenius base changes. And using

this result we will also see the  $p$ -rank 0 primes for a certain non-closed point in the moduli spaces of curves over a number field is finite.

### **LEE Dong Uk**

**Title:** Nonemptiness of the  $\mu$ -ordinary locus of Shimura varieties of Hodge type

**Abstract:** We prove that every Shimura variety of Hodge type has nonempty  $\mu$ -ordinary locus at the primes of its reflex field where the group is unramified. We also verify a nonemptiness criterion of the ordinary locus for Hodge-type Shimura varieties.

### **MOON Hyunsuk**

**Title:** On the structure of the Mordell-Weil groups of Jacobians over infinite number fields

**Abstract:** Frey and Jarden have asked whether the Mordell-Weil group of every nonzero abelian variety defined over a number field  $K$  has infinite Mordell-Weil rank over the maximal abelian extension of  $K$ . Rosen and Wong proved the infiniteness of the rank for the Jacobian of any curve that can be realized over  $K$  as a cyclic geometrically irreducible cover of the projective line. In this talk, we will give another proof of Rosen-Wong's result together with slightly more precise information on the structure of the Mordell-Weil group.

### **NAKAMURA Kentaro**

**Title:** Classification of 2-dimensional split trianguline representations of  $p$ -adic fields

**Abstract:** Split trianguline representation is a class of  $p$ -adic representations of Galois groups of  $p$ -adic fields, which plays essential roles in Colmez's  $p$ -adic local Langlands correspondence for  $\mathrm{GL}_2(\mathbb{Q}_p)$ . In my talk, I will completely classify 2-dimensional split trianguline representations for any  $p$ -adic fields.

### **OTSUKI Rei**

**Title:** A homomorphism concerning two systems for an elliptic curve

**Abstract:** There are some results about the Selmer groups in Iwasawa theory for elliptic curves with supersingular reduction, using the correspondence between the zeta elements defined by Kato and the modular elements defined by Mazur-Tate through a certain homomorphism. We will talk about the generalization of the homomorphism and its application.

### **SUN Hae-Sang**

**Title:** A cuspidal class number of the tower of modular curves

**Abstract:** In this talk, we discuss  $p$ -adic and  $\ell$ -adic valuations of the first type cuspidal class number of the modular curves  $X_0(Np^n)$ ,  $n = 1, 2, 3, \dots$ . Main ingredients are the formula of Kubert-Lang-Yu and an extension of a Washington's theorem to the generalized Bernoulli number of arbitrary degree.

#### **YAMAZAKI Takao**

**Title:** Counter examples to variants of the Milnor-Bloch-Kato conjecture

**Abstract:** Galois symbol on semi-abelian varieties has been conjectured to be injective, as a generalization of the Milnor-Bloch-Kato conjecture. After a review on its role in higher dimensional class field theory, I will present a counter example to the injectivity conjecture, and discuss its relation to the theory of motivic cohomology. (Joint work with Michael Spiess.)

#### **YOSHIDA Manabu**

**Title:** Ramification of local fields and Fontaine's property  $(P_m)$

**Abstract:** Let  $K$  be a complete discrete valuation field with perfect residue field. Consider the ramification filtration  $(G^j)$  in the upper numbering of the Galois group of a finite Galois extension  $L$  of  $K$ . Then Fontaine characterized the greatest break of the ramification filtration by a certain property  $(P_m)$  of the extension  $L/K$  for real numbers  $m$ . By refining Fontaine's result, we obtain a new interpretation of the ramification filtration in terms of the property  $(P_m)$ .