The Fault-Tolerant Facility Location Problem with Submodular Penalties

Naoyuki Kamiyama

MI 2013-15

(Received December 8, 2013)
The Fault-Tolerant Facility Location Problem with Submodular Penalties

Naoyuki Kamiyama*

Institute of Mathematics for Industry, Kyushu University
kamiyama@imi.kyushu-u.ac.jp

Abstract
In this paper, we consider the fault-tolerant facility location problem with submodular penalties that is a common generalization of the fault-tolerant facility location problem and the facility location problem with submodular penalties. For this problem, we present a combinatorial $3 \cdot H_R$-approximation algorithm, where $R$ is the maximum connectivity requirement and $H_R$ is the $R$-th harmonic number. Our algorithm is a common generalization of the algorithm for the the fault-tolerant facility location problem presented by Jain and Vazirani (2003) and that for the facility location problem with submodular penalties presented by Du, Lu and Xu (2012).

1 Introduction

The facility location problem is one of the most important problems in combinatorial optimization. Unfortunately, this problem is NP-hard, and thus most of the research has been focusing on designing approximation algorithms with good performance. So far, the best approximation ratio for the facility location problem is $1.488$ due to Li [8].

Many variants of the facility location problem have appeared. The fault-tolerant facility location problem is one of variants of the facility location problem. In this problem, each client has a connectivity requirement and we have to connect each client to as many open facilities as its connectivity requirement. This problem was introduced by Jain and Vazirani [7], and several approximation algorithms were presented [7, 5, 11, 1]. So far, the best approximation ratio for the fault-tolerant facility location problem is $1.725$ due to Byrka, Sinrivasan and Swamy [1].

The facility location problem with submodular penalties is another variant of the facility location problem. In this problem, not all clients are connected to open facilities and unconnected clients incur a penalty cost determined by a monotone submodular function on the client set. This problem was introduced by Hayrapetyan, Swamy and Tardos [6], and several approximation algorithms were presented [6, 2, 3, 9]. So far, the best approximation ratio for the facility location problem with submodular penalties is $2$ due to Li, Du, Xiu and Xu [9].

In this paper, we consider the fault-tolerant facility location problem with submodular penalties that is a common generalization of the above two variants of the facility location problem. For this problem, we present a combinatorial $3 \cdot H_R$-approximation algorithm, where $R$ is the maximum connectivity requirement and $H_R$ is the $R$-th harmonic number. Our algorithm is a common generalization of the algorithm for the the fault-tolerant facility location problem presented by Jain and Vazirani [7] and that for the facility location problem with submodular penalties presented by Du, Lu and Xu [3].

*This work is partly supported by KAKENHI(24106005).
Organization. In Section 2, we give a formal definition of the fault-tolerant facility location problem with submodular penalties. In Section 3, we present an algorithm for the fault-tolerant facility location problem with submodular penalties. In Section 4, we analyze an approximation ratio of our algorithm. Section 5 concludes this paper.

Notation. We denote by $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{Z}_+$ the sets of real numbers, non-negative real numbers and non-negative integers, respectively.

Assume that we are given a set $U$. For each subset $X$ of $U$, we define a characteristic vector $\chi_X$ in $\{0,1\}^U$ by

$$\chi_X(u) := \begin{cases} 1 & \text{if } u \in X \\ 0 & \text{if } u \in U \setminus X. \end{cases}$$

Let $d_1, d_2$ be vectors in $\mathbb{R}^U$. Define a vector $d_1 \pm d_2$ in $\mathbb{R}^U$ by

$$(d_1 \pm d_2)(u) := d_1(u) \pm d_2(u).$$

In addition, we write $d_1 \succeq d_2$, if $d_1(u) \geq d_2(u)$ for every element $u$ in $U$.

2 Problem Formulation

The fault-tolerant facility location problem with submodular penalties is defined as follows. We are given a finite set $F$ of facilities and a finite set $D$ of clients. For each facility $i$ in $F$, an opening cost $f_i$ in $\mathbb{R}_+$ is given. For each client $j$ in $D$, a connectivity requirement $r_j$ in $\mathbb{Z}_+$ is given. We assume that $r_j \leq |F|$ for every client $j$ in $D$. For each facility $i$ in $F$ and each client $j$ in $D$, a connecting cost $c_{i,j}$ in $\mathbb{R}_+$ is given. We assume that connecting costs satisfy the triangle inequality, i.e.,

$$c_{i,j} \geq c_{i,j'} + c_{i',j}$$

for every facilities $i, i'$ in $F$ and every clients $j, j'$ in $D$. In addition, we are given a penalty function $h: 2^D \to \mathbb{R}_+$, which is the Lovász extension of a non-negative monotone submodular function with $h(\emptyset) = 0$. We will give formal definitions of a submodular function and its Lovász extension later.

An assignment is a triple $(X,d,\varphi)$ of a subset $X$ of $F$, functions $d: D \to \mathbb{Z}_+$ and $\varphi: D \to 2^F$. An assignment is said to be feasible, if

$$\forall j \in D: \varphi(j) \subseteq X \text{ and } |\varphi(j)| + d(j) = r_j.$$  

The cost $\xi(X,d,\varphi)$ of an assignment $(X,d,\varphi)$ is defined by

$$\xi(X,d,\varphi) := \sum_{i \in X} f_i + \sum_{j \in D} \sum_{i \in \varphi(j)} c_{i,j} + h(d).$$

The goal of the fault-tolerant facility location problem with submodular penalties is to find a feasible assignment with minimum cost.

Here we give formal definitions of a submodular function and its Lovász extension. A function $\rho: 2^D \to \mathbb{R}_+$ is said to be submodular, if

$$\forall X,Y \subseteq D: \rho(X) + \rho(Y) \geq \rho(X \cup Y) + \rho(X \cap Y).$$  

(1)

It is well known that (1) is equivalent to

$$\forall X,Y \subseteq D \text{ s.t. } X \subseteq Y, \forall j \in D \setminus Y: \rho(X \cup \{j\}) - \rho(X) \geq \rho(Y \cup \{j\}) - \rho(Y).$$  

(2)
A submodular function $\rho: \mathcal{P} \rightarrow \mathbb{R}_+$ is said to be monotone, if
\[
\forall X, Y \subseteq D \text{ s.t. } X \subseteq Y : \rho(X) \leq \rho(Y).
\]
The Lovász extension $h: \mathcal{P} \rightarrow \mathbb{R}_+$ of a submodular function $\rho: \mathcal{P} \rightarrow \mathbb{R}_+$ is defined as follows. Assume that we are given a vector $d$ in $\mathbb{R}_+^D$. We denote by $\hat{d}_1 > \hat{d}_2 > \cdots > \hat{d}_k$ the distinct values of its components and define
\[
U_l := \{ j \in D \mid d(j) \geq \hat{d}_l \}
\]
for each $l = 1, 2, \ldots, k$. We define $h(d)$ by
\[
h(d) := \sum_{l=1}^{k-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \hat{d}_k \cdot \rho(U_k).
\]
It is not difficult to see that $\rho(X) = h(\chi_X)$ for every subset $X$ of $D$. This implies that the fault-tolerant facility location problem with submodular penalties is a generalization of the facility location problem with submodular penalties.

Here we give properties of the function $h$ that will be used in the sequel. It is known [4] that $h(d)$ is equal to the optimal value of the following linear programming (4).
\[
\begin{aligned}
\max & \quad \sum_{j \in D} d(j) \cdot p_j \\
\text{s.t.} & \quad \sum_{j \in X} p_j \leq \rho(X) \quad (X \subseteq D) \\
& \quad p_j \in \mathbb{R} \quad (j \in D).
\end{aligned}
\tag{4}
\]
In addition, it follows from the monotonicity of $\rho$ that $h$ is also “monotone”.

**Lemma 1.** For every vectors $d, d'$ in $\mathbb{Z}_+^D$ with $d \geq d'$, we have $h(d) \geq h(d')$.

**Proof.** We prove this lemma by induction on $\|d\| := \sum_{j \in D} d(j)$.

If $\|d\| = 0$, then $h(d) \geq h(d')$ clearly holds for every vector $d'$ in $\mathbb{Z}_+^D$ with $d \geq d'$. Assuming that this lemma holds for every vector $d$ in $\mathbb{Z}_+^D$ with $\|d\| = \Delta$, we consider the case of $\|d\| = \Delta + 1$.

If we can prove that
\[
\forall j \in D \text{ s.t. } d(j) > 0 : h(d) \geq h(d - \chi_{\{j\}}),
\]
then this lemma follows from the induction hypothesis. Let us fix a client $j$ in $D$ with $d(j) > 0$. We denote by $\hat{d}_1 > \hat{d}_2 > \cdots > \hat{d}_k$ the distinct values of the components of $d$ and define $U_l$ by (3) for each $l = 1, 2, \ldots, k$. Assume that $d(j) = \hat{d}_s$. We have
\[
h(d - \chi_{\{j\}}) := \sum_{l=1}^{s-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \rho(U_s \setminus \{j\}) + (\hat{d}_s - \hat{d}_{s+1} - 1) \rho(U_s)
\]
\[
+ \sum_{l=s+1}^{k-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \hat{d}_k \cdot \rho(U_k).
\]
Hence, we have
\[
h(d) - h(d - \chi_{\{j\}}) = \rho(U_s) - \rho(U_s \setminus \{j\}) \geq 0,
\]
where the inequality follows from the monotonicity of $\rho$. This completes the proof. \qed
3 Algorithm

In this section, we explain our algorithm for the fault-tolerant facility location problem with submodular penalties. For this, we first define a subproblem \( S(H, K, \sigma) \) for each subset \( H \) of \( F \), each subset \( K \) of \( D \) and each function \( \sigma: K \to 2^H \) as follows. Intuitively, in \( S(H, K, \sigma) \), facilities in \( H \) are already open, and each client \( j \) of \( K \) is not allowed to be connected to facilities in \( \sigma(j) \). Under this constraint, this subproblem asks for opening facilities in \( F \setminus H \) and connecting each client \( j \) in \( K \) to a newly opened facility or a facility in \( H \setminus \sigma(j) \). Notice that in this problem, a connectivity requirement of each client in \( K \) is equal to one.

A feasible solution of \( S(H, K, \sigma) \) is a triple \((Y, P, \psi)\) of a subset \( Y \) of \( F \setminus H \), a subset \( P \) of \( K \) and a function \( \psi: K \setminus P \to F \) such that

\[
\forall j \in K \setminus P: \psi(j) \in (H \cup Y) \setminus \sigma(j).
\]

The cost \( \xi_S(Y, P, \psi) \) of a feasible solution \((Y, P, \psi)\) of \( S(H, K, \sigma) \) is defined by

\[
\xi_S(Y, P, \psi) := \sum_{i \in Y} f_i + \sum_{j \in K \setminus P} c_{\psi(j), j} + \rho(P).
\]

The goal of \( S(H, K, \sigma) \) is to find a feasible solution with minimum cost.

Now we are ready to present our algorithm for the fault-tolerant facility location problem with submodular penalties, called \( \text{FTFLwSP} \). Define

\[ R := \max\{r_j \mid j \in D\}. \]

The algorithm \( \text{FTFLwSP} \) is described as follows.

**Step 1:** Set \( t := R, X_{t+1} := \emptyset, d_{t+1}(j) := 0 \) and \( \varphi_{t+1}(j) := \emptyset \) for each client \( j \) in \( D \).

**Step 2:** If \( t \geq 1 \), do the following (2-a) to (2-d).

- **(2-a)** Set \( H_t := X_{t+1}, K_t := \{j \in D \mid r_j \geq t\} \) and \( \sigma_t(j) := \varphi_{t+1}(j) \) for each client \( j \) in \( K_t \).
- **(2-b)** Find a feasible solution \((Y_t, P_t, \psi_t)\) of \( S(H_t, K_t, \sigma_t) \).
- **(2-c)** Set \( X_t := X_{t+1} \cup Y_t, d_t := d_{t+1} + \chi_{P_t}, \) and

\[
\varphi_t(j) := \begin{cases} 
\varphi_{t+1}(j) \cup \{\psi_t(j)\} & \text{if } j \in K_t \setminus P_t \\
\varphi_{t+1}(j) & \text{if } j \in (D \setminus K_t) \cup P_t.
\end{cases}
\]

- **(2-d)** Update \( t := t - 1 \) and go to **Step 2**.

**Step 3:** Output \((X_1, d_1, \varphi_1)\).

In **Step 2** of the \( t \)-th iteration, we connect a client \( j \) in \( D \) with \( r_j \geq t \) to some open facility or increase its penalty by one. Hence, the output \((X_1, d_1, \varphi_1)\) is clearly a feasible assignment. Notice that an approximation ratio of the algorithm \( \text{FTFLwSP} \) depends on the quality of \((Y_t, P_t, \psi_t)\) in **Step (2-b)**. In addition, if we can find \((Y_t, P_t, \psi_t)\) in polynomial time, then the algorithm \( \text{FTFLwSP} \) is also a polynomial-time algorithm. We will discuss these points in the next section.
4 Analysis

In this section, we analyze an approximation ratio of the algorithm FTFLwSP. An IP formulation of the fault-tolerant facility location problem with submodular penalties is described as follows.

\[
\begin{align*}
\min & \quad \sum_{i \in F} f_i y_i + \sum_{i \in F} \sum_{j \in D} c_{i,j} x_{i,j} + h(d) \\
\text{s.t.} & \quad \sum_{i \in F} x_{i,j} + d(j) \geq r_j \quad (j \in D) \\
& \quad x_{i,j} \leq y_i \quad (i \in F, j \in D) \\
& \quad x_{i,j} \in \{0,1\} \quad (i \in F, j \in D) \\
& \quad y_i \in \{0,1\} \quad (i \in F) \\
& \quad d \in \mathbb{Z}_+^N.
\end{align*}
\]

Notice that it follows from Lemma 1 that there exists an optimal solution of (5) such that the first constraint holds with equality for every client \(j\) in \(D\). Denote by \(\text{OPT}\) the optimal value of (5), i.e., the fault-tolerant facility location problem with submodular penalties.

Here we consider an LP relaxation of (5). The dual problem of (4) is described as follows.

\[
\begin{align*}
\min & \quad \sum_{X \subseteq D} \rho(X) \cdot q_X \\
\text{s.t.} & \quad \sum_{X \subseteq D: j \in X} q_X = d(j) \quad (j \in D) \\
& \quad q_X \geq 0 \quad (X \subseteq D).
\end{align*}
\]

It follows from (6) that an LP relaxation of (5) can be described as follows.

\[
\begin{align*}
\min & \quad \sum_{i \in F} f_i y_i + \sum_{i \in F} \sum_{j \in D} c_{i,j} x_{i,j} + \sum_{X \subseteq D} \rho(X) \cdot q_X \\
\text{s.t.} & \quad \sum_{i \in F} x_{i,j} + \sum_{X \subseteq D: j \in X} q_X \geq r_j \quad (j \in D) \\
& \quad x_{i,j} \leq y_i \quad (i \in F, j \in D) \\
& \quad x_{i,j} \geq 0 \quad (i \in F, j \in D) \\
& \quad 0 \leq y_i \leq 1 \quad (i \in F) \\
& \quad q_X \geq 0 \quad (X \subseteq D).
\end{align*}
\]

Denote by \(\text{OPT}_{\text{LP}}\) the optimal value of (7). Notice that \(\text{OPT}_{\text{LP}} \leq \text{OPT}\) holds. The dual problem of (7) is described as follows.

\[
\begin{align*}
\max & \quad \sum_{j \in D} r_j \alpha_j - \sum_{i \in F} z_i \\
\text{s.t.} & \quad \alpha_j - \beta_{i,j} \leq c_{i,j} \quad (i \in F, j \in D) \\
& \quad \sum_{j \in D} \beta_{i,j} \leq f_i + z_i \quad (i \in F) \\
& \quad \sum_{j \in X} \alpha_j \leq \rho(X) \quad (X \subseteq D) \\
& \quad \alpha_j \geq 0 \quad (j \in D) \\
& \quad \beta_{i,j} \geq 0 \quad (i \in F, j \in D) \\
& \quad z_i \geq 0 \quad (i \in F).
\end{align*}
\]
Next we consider an IP formulation of $S(H_t, K_t, \sigma_t)$. Define a new function $\rho_{K_t} : 2^{K_t} \to \mathbb{R}_+$ by $\rho_{K_t}(X) := \rho(X)$ for each subset $X$ of $K_t$. It is clear that $\rho_{K_t}$ is submodular. Let $h_{K_t} : \mathbb{R}^{K_t}_+ \to \mathbb{R}_+$ be the Lovász extension of $\rho_{K_t}$. An IP formulation of $S(H_t, K_t, \sigma_t)$ is described as follows.

$$
\min \sum_{i \in F \setminus H_t} f_i y_i + \sum_{i \in F \setminus K_t} \sum_{j \in K_t} c_{i,j} x_{i,j} + h_{K_t}(d)
$$

s.t.
$$
\sum_{i \in F \setminus \sigma_t(j)} x_{i,j} + d(j) \geq 1 \quad (j \in K_t)
$$

$$
x_{i,j} \leq y_i \quad (i \in F \setminus H_t, j \in K_t)
$$

$$
x_{i,j} \in \{0, 1\} \quad (i \in F, j \in K_t)
$$

$$
y_i \in \{0, 1\} \quad (i \in F \setminus H_t)
$$

$$
d \in \{0, 1\}^{K_t}. \tag{9}
$$

Similarly to (7), an LP relaxation of (9) is described as follows.

$$
\min \sum_{i \in F \setminus H_t} f_i y_i + \sum_{i \in F \setminus K_t} \sum_{j \in K_t} c_{i,j} x_{i,j} + \sum_{X \subseteq K_t} \rho_{K_t}(X) \cdot q_X
$$

s.t.
$$
\sum_{i \in F \setminus \sigma_t(j)} x_{i,j} \geq 1 \quad (j \in K_t)
$$

$$
x_{i,j} \leq y_i \quad (i \in F \setminus H_t, j \in K_t)
$$

$$
x_{i,j} \geq 0 \quad (i \in F \setminus H_t, j \in K_t)
$$

$$
y_i \geq 0 \quad (i \in F \setminus H_t)
$$

$$
q_X \geq 0 \quad (X \subseteq K_t). \tag{10}
$$

Denote by $\text{OPT}_{SLP}(t)$ the optimal value of (10). The dual problem of (10) is described as follows.

$$
\max \sum_{i \in K_t} \alpha_i
$$

s.t.
$$
\alpha_j - \beta_{i,j} \leq c_{i,j} \quad (j \in K_t, i \in F \setminus H_t)
$$

$$
\alpha_j \leq c_{i,j} \quad (j \in K_t, i \in H_t \setminus \sigma_t(j))
$$

$$
\sum_{j \in K_t} \beta_{i,j} \leq f_i \quad (i \in F \setminus H_t)
$$

$$
\sum_{j \in X} \alpha_j \leq \rho_{K_t}(X) \quad (X \subseteq K_t)
$$

$$
\alpha_j \geq 0 \quad (j \in K_t)
$$

$$
\beta_{i,j} \geq 0 \quad (i \in F \setminus H_t, j \in K_t). \tag{11}
$$

From now on, we analyze an approximation ratio of the algorithm $\text{FTFLwSP}$.

**Lemma 2.** For every $t = 1, 2, \ldots, R$, we can find a feasible solution $(Y_t, P_t, \psi_t)$ of $S(H_t, K_t, \sigma_t)$ such that

$$
\xi_S(Y_t, P_t, \psi_t) \leq 3 \cdot \text{OPT}_{SLP}(t)
$$

in polynomial time.

We will give the proof of Lemma 2 in the next subsection.

**Lemma 3.** For every $t = 1, 2, \ldots, R$, we have

$$
\text{OPT}_{SLP}(t) \leq \frac{1}{t} \cdot \text{OPT}_{LP}.
$$
Proof. It follow from the strong duality theorem that there exists a feasible solution
\[ \alpha_j \ (j \in K_t), \quad \beta_{i,j} \ (i \in F \setminus H_t; \ j \in K_t) \]
of (11) with
\[ \sum_{j \in K_t} \alpha_j = \text{OPT}_{\text{SLP}}(t). \]

To prove the theorem, we construct a feasible solution
\[ \hat{\alpha}_j \ (j \in D), \quad \hat{\beta}_{i,j} \ (i \in F; \ j \in D), \quad \hat{z}_i \ (i \in F) \]
of (8) with
\[ \sum_{j \in D} r_j \hat{\alpha}_j - \sum_{i \in F} \hat{z}_i \geq t \cdot \text{OPT}_{\text{SLP}}(t). \]

It follows from the weak duality theorem that
\[ \sum_{j \in D} r_j \hat{\alpha}_j - \sum_{i \in F} \hat{z}_i \leq \text{OPT}_{\text{LP}}, \]
which completes the proof.

We first define \( \hat{\alpha}_j \) for each client \( j \) in \( D \) by
\[ \hat{\alpha}_j := \begin{cases} \alpha_j & \text{if } j \in K_t \\ 0 & \text{if } j \in D \setminus K_t. \end{cases} \]

Next we define \( \hat{\beta}_{i,j} \) for each facility \( i \) in \( F \) and each client \( j \) in \( D \) by
\[ \hat{\beta}_{i,j} := \begin{cases} \beta_{i,j} & \text{if } j \in K_t \text{ and } i \in F \setminus H_t \\ 0 & \text{if } j \in K_t \text{ and } i \in H_t \setminus \sigma_t(j) \\ \alpha_j & \text{if } j \in K_t \text{ and } i \in \sigma_t(j) \\ 0 & \text{if } j \in D \setminus K_t \text{ and } i \in F. \end{cases} \]

Finally, we define \( \hat{z}_i \) for each facility \( i \) in \( F \) by
\[ \hat{z}_i := \begin{cases} \sum_{j \in K_t: i \in \sigma_t(j)} \alpha_j & \text{if } i \in H_t \\ 0 & \text{if } i \in F \setminus H_t. \end{cases} \]

Here we prove that \( \hat{\alpha}_i, \hat{\beta}_{i,j} \) and \( \hat{z}_i \) are a feasible solution of (8). We first consider the first constraint. For each client \( j \) in \( K_t \) and each facility \( i \) in \( F \setminus H_t \),
\[ \hat{\alpha}_j - \hat{\beta}_{i,j} = \alpha_j - \beta_{i,j} \leq c_{i,j}. \]

For each client \( j \) in \( K_t \) and each facility \( i \) in \( H_t \setminus \sigma_t(j) \),
\[ \hat{\alpha}_j - \hat{\beta}_{i,j} = \alpha_j - 0 = \alpha_j \leq c_{i,j}. \]

For each client \( j \) in \( K_t \) and each facility \( i \) in \( \sigma_t(j) \),
\[ \hat{\alpha}_j - \hat{\beta}_{i,j} = \alpha_j - \alpha_j = 0 \leq c_{i,j}. \]
For each client \( j \) in \( D \setminus K_t \) and each facility \( i \) in \( F \),
\[
\hat{\alpha}_j - \hat{\beta}_{i,j} = 0 - 0 = 0 = c_{i,j}.
\]

Next we consider the second constraint. For each facility \( i \) in \( H_t \),
\[
\sum_{j \in D} \hat{\beta}_{i,j} = \sum_{j \in K_t} \sum_{j \in K_t : i \in \sigma_t(j)} \alpha_j = \hat{z}_i \leq f_i + \hat{z}_i.
\]

For each facility \( i \) in \( F \setminus H_t \),
\[
\sum_{j \in D} \hat{\beta}_{i,j} = \sum_{j \in K_t} \beta_{i,j} \leq f_i \leq f_i + \hat{z}_i.
\]

Finally, we consider the third constraint. For each subset \( X \) of \( K_t \).
\[
\sum_{j \in X} \hat{\alpha}_j = \sum_{j \in X} \alpha_j \leq \rho_{K_t}(X) = \rho(X).
\]

For each subset \( X \) of \( D \) with \( X \setminus K_t \neq \emptyset \),
\[
\sum_{j \in X} \hat{\alpha}_j = \sum_{j \in X} \alpha_j \leq \rho_{K_t}(X \cap K_t) = \rho(X \cap K_t) \leq \rho(X),
\]

where the last inequality follows from the monotonicity of \( \rho \).

Next we consider the objective value.
\[
\sum_{j \in D} r_j \hat{\alpha}_j - \sum_{i \in F} \hat{z}_i = \sum_{j \in K_t} \sum_{i \in H_t \cup K_t} \sum_{i \in \sigma_t(j)} \alpha_j
\]
\[
= \sum_{j \in K_t} r_j \alpha_j - \sum_{i \in H_t \cup K_t : i \in \sigma_t(j)} \alpha_j
\]
\[
= \sum_{j \in K_t} r_j \alpha_j - \sum_{j \in K_t} |\sigma_t(j)| \alpha_j
\]
\[
= \sum_{j \in K_t} (r_j - |\varphi_{t+1}(j)|) \alpha_j \quad \text{(by } \sigma_t(j) = \varphi_{t+1}(j))
\]
\[
\geq \sum_{j \in K_t} (r_j - (r_j - t)) \alpha_j \quad \text{(by } |\varphi_{t+1}(j)| \leq r_j - t)
\]
\[
= \sum_{j \in K_t} t \cdot \alpha_j
\]
\[
= t \cdot \text{OPT}_{\text{SLP}}(t).
\]

This completes the proof. \( \square \)

**Lemma 4.** For every vector \( d \) in \( \mathbb{Z}_+^D \) and every subset \( X \) of \( D \), we have
\[
h(d + \chi X) - h(d) \leq h(\chi X) = \rho(X). \tag{12}
\]

**Proof.** If \( d(j) = 0 \) for every client \( j \) in \( D \), then (12) clearly holds. Assume that there exists a client \( j \) in \( D \) with \( d(j) > 0 \). To prove (12), it suffices to prove that there exists a client \( j^* \) in \( D \) such that \( d(j^*) > 0 \) and
\[
h(d + \chi X) - h(d) \leq h(d^* + \chi X) - h(d^*), \tag{13}
\]
where the vector \( d^* \) in \( \mathbb{R}^D_+ \) is define by \( d^* := d - \chi_{\{j^*\}} \).

We denote by \( d_1 > d_2 > \cdots > d_k \) the distinct values of the components of \( d \) and define

\[
U_l := \{ j \in D \mid d(j) \geq \hat{d}_l \} \\
X_l := \{ j \in X \mid d(j) = \hat{d}_l \} \\
\overline{X}_l := \{ j \in D \setminus X \mid d(j) = \hat{d}_l \} \\
U^+_l := U_{l-1} \cup \overline{X}_l
\]

for each \( l = 1, 2, \ldots, k \), where define \( U_0 := \emptyset \). Define \( j^* \) in \( D \) by

\[
j^* := \begin{cases} 
\text{a client in } X_k & \text{if } \hat{d}_k \neq 0 \text{ and } X_k \neq \emptyset \\
\text{a client in } X_{k-1} & \text{if } \hat{d}_k = 0 \text{ and } X_{k-1} \neq \emptyset \\
\text{a client in } X_{k-1} & \text{if } \hat{d}_k = 0 \text{ and } \overline{X}_{k-1} = \emptyset \\
\end{cases}
\]

First we calculate the left-hand side of (13). Since

\[
h(d) = \sum_{l=1}^{k-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \hat{d}_k \cdot \rho(U_k)
\]

\[
h(d + \chi_X) = \sum_{l=1}^{k-1} \left( \rho(U^+_l) + (\hat{d}_l - \hat{d}_{l+1} - 1)\rho(U_l) \right) + \rho(U^+_k) + \hat{d}_k \cdot \rho(U_k),
\]

we have

\[
h(d + \chi_X) - h(d) = \sum_{l=1}^{k-1} \left( \rho(U^+_l) - \rho(U_l) \right) + \rho(U^+_k).
\]

Next we consider the right-hand side of (13). Assume that \( \hat{d}_k \neq 0 \) and \( \overline{X}_k \neq \emptyset \). In this case,

\[
h(d^*) = \sum_{l=1}^{k-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \rho(U_k \setminus \{ j^* \}) + (\hat{d}_k - 1) \cdot \rho(U_k)
\]

\[
h(d^* + \chi_X) = \sum_{l=1}^{k-1} \left( \rho(U^+_l) + (\hat{d}_l - \hat{d}_{l+1} - 1)\rho(U_l) \right) \\
+ \rho(U^+_k) + \rho(U_k \setminus \{ j^* \}) + (\hat{d}_k - 1) \cdot \rho(U_k).
\]

Hence, we have

\[
h(d^* + \chi_X) - h(d^*) = \sum_{l=1}^{k-1} \left( \rho(U^+_l) - \rho(U_l) \right) + \rho(U^+_k),
\]

which implies that (13) holds.

Assume that \( \hat{d}_k \neq 0 \) and \( \overline{X}_k = \emptyset \). In this case,

\[
h(d^*) = \sum_{l=1}^{k-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \rho(U_k \setminus \{ j^* \}) + (\hat{d}_k - 1) \cdot \rho(U_k)
\]

\[
h(d^* + \chi_X) = \sum_{l=1}^{k-1} \left( \rho(U^+_l) + (\hat{d}_l - \hat{d}_{l+1} - 1)\rho(U_l) \right) + \rho(U^+_k \setminus \{ j^* \}) + \hat{d}_k \cdot \rho(U^+_k).
\]
Hence, since \( U_k = U_k^+ \), we have
\[
h(d^* + \chi X) - h(d^*) = \sum_{l=1}^{k-1} \left( \rho(U_l^+) - \rho(U_l) \right) + \rho(U_k^+),
\]
which implies that (13) holds.

Assume that \( \hat{d}_k = 0 \) and \( \overline{X}_{k-1} \neq \emptyset \). We first consider the case of \( \hat{d}_{k-1} > 1 \). In this case,
\[
\begin{align*}
h(d^*) &= \sum_{l=1}^{k-2} (d_l - \hat{d}_{l+1}) \rho(U_l) + \rho(U_{k-1} \setminus \{j^*\}) + (\hat{d}_{k-1} - 1) \cdot \rho(U_{k-1}) \\
h(d^* + \chi X) &= \sum_{l=1}^{k-2} \left( \rho(U_l^+) + (d_l - \hat{d}_{l+1} - 1) \rho(U_l) \right) \\
&\quad + \rho(U_{k-1}^+) + \rho(U_{k-1} \setminus \{j^*\}) + (\hat{d}_{k-1} - 2) \cdot \rho(U_{k-1}) + \rho(U_k^+).
\end{align*}
\]
Hence, we have
\[
h(d^* + \chi X) - h(d^*) = \sum_{l=1}^{k-1} \left( \rho(U_l^+) - \rho(U_l) \right) + \rho(U_k^+),
\]
which implies that (13) holds. If \( \hat{d}_{k-1} = 1 \), then we have
\[
\begin{align*}
h(d^*) &= \sum_{l=1}^{k-2} (d_l - \hat{d}_{l+1}) \rho(U_l) + \rho(U_{k-1} \setminus \{j^*\}) \\
h(d^* + \chi X) &= \sum_{l=1}^{k-2} \left( \rho(U_l^+) + (d_l - \hat{d}_{l+1} - 1) \rho(U_l) \right) + \rho(U_{k-1}^+) + \rho(U_k^+) \setminus \{j^*\}).
\end{align*}
\]
Hence, we have
\[
\begin{align*}
h(d^* + \chi X) - h(d^*) &= \sum_{l=1}^{k-1} \left( \rho(U_l^+) - \rho(U_l) \right) + \rho(U_{k-1}^+) + \rho(U_k^+) \setminus \{j^*\}) - \rho(U_{k-1} \setminus \{j^*\}) \\
&\geq \sum_{l=1}^{k-1} \left( \rho(U_l^+) - \rho(U_l) \right) + \rho(U_k^+),
\end{align*}
\]
where the inequality follows from (2) and \( U_{k-1} \subseteq U_k^+ \). This implies that (13) holds.

Assume that \( \hat{d}_k = 0 \) and \( \overline{X}_{k-1} = \emptyset \). In this case,
\[
\begin{align*}
h(d^*) &= \sum_{l=1}^{k-2} (d_l - \hat{d}_{l+1}) \rho(U_l) + \rho(U_{k-1} \setminus \{j^*\}) + (\hat{d}_{k-1} - 1) \cdot \rho(U_{k-1}) \\
h(d^* + \chi X) &= \sum_{l=1}^{k-2} \left( \rho(U_l^+) + (d_l - \hat{d}_{l+1} - 1) \rho(U_l) \right) \\
&\quad + \rho(U_{k-1}^+) \setminus \{j^*\}) + (\hat{d}_{k-1} - 1) \rho(U_{k-1}^+) + \rho(U_k^+).
\end{align*}
\]
Hence, since \( U_{k-1} = U_{k-1}^+ \), we have
\[
\begin{align*}
h(d^* + \chi X) - h(d^*) &= \sum_{l=1}^{k-2} \left( \rho(U_l^+) - \rho(U_l) \right) + \rho(U_k^+) \\
&= \sum_{l=1}^{k-1} \left( \rho(U_l^+) - \rho(U_l) \right) + \rho(U_k^+).
\end{align*}
\]
This completes the proof. □

Now we are ready to prove the main result of this paper.

**Theorem 5.** We have

\[ \xi(X_1, d_1, \varphi_1) \leq 3 \cdot H_R \cdot \text{OPT}, \]

where \(H_R := 1 + \frac{1}{2} + \cdots + \frac{1}{R}.\)

**Proof.** It follows from Lemma 4 that

\[ \forall t = 1, \ldots, R: \xi(X_t, d_t, \varphi_t) - \xi(X_t+1, d_{t+1}, \varphi_{t+1}) \leq \xi_S(Y_t, P_t, \psi_t). \quad (14) \]

Thus, it follows from (14) that

\[ \xi(X_1, d_1, \varphi_1) \leq \sum_{t=1}^{R} \xi_S(Y_t, P_t, \psi_t). \quad (15) \]

Hence, it follows from (15) and Lemmas 3, 4 that

\[ \xi(X_1, d_1, \varphi_1) \leq \sum_{t=1}^{R} 3 \cdot \text{OPT}_{\text{SLP}}(t) \leq 3 \cdot H_R \cdot \text{OPT}_{\text{LP}} \leq 3 \cdot H_R \cdot \text{OPT}. \]

This completes the proof. □

### 4.1 Proof of Lemma 2

In this subsection, we prove Lemma 2. Our algorithm for \(S(H_t, K_t, \sigma_t)\) is essentially the same as the algorithm proposed by Du, Lu and Xu [3] except the following differences.

- The opening cost of a facility in \(H_t\) is zero.
- The connection cost between a client \(j\) in \(K_t\) and a facility \(i\) in \(\sigma_t(j)\) is infinite.

For completeness, we reproduce the algorithm of Du, Lu and Xu [3] in our setting. Define a modified opening cost \(\hat{f}_i\) for each facility \(i\) in \(F\) by

\[ \hat{f}_i := \begin{cases} f_i & \text{if } i \in F \setminus H_t \\ 0 & \text{if } i \in H_t. \end{cases} \]

Define a modified connecting cost \(\hat{c}_{i,j}\) for each facility \(i\) in \(F\) and each client \(j\) in \(K_t\) by

\[ \hat{c}_{i,j} := \begin{cases} c_{i,j} & \text{if } i \in F \setminus \sigma(j) \\ \infty & \text{if } i \in \sigma(j). \end{cases} \]

Notice that connecting costs do not necessarily satisfy the triangle inequality.

Our algorithm consists of two phases. In the first phase, we use a concept of time \(\delta\). The algorithm starts at \(\delta = 0\). Initially, we set \(\alpha_j := 0\) for each client \(j\) in \(K_t\) and \(\beta_{i,j} := 0\) for each facility \(j\) in \(F \setminus H_t\) and each client \(j\) in \(K_t\). Facilities \(i\) in \(F\) with \(\hat{f}_i > 0\) are closed, and facilities \(i\) in \(F\) with \(\hat{f}_i = 0\) are open. Every client in \(K_t\) is unfrozen. Let \(P\) be the set of penalized client, and set \(P := \emptyset\).

The algorithm increases \(\alpha_j\) for all unfrozen clients \(j\) in \(K_t\) uniformly at the unit rate \(\delta\), and declares the pair \((i, j)\) of a facility \(i\) in \(F\) and a client \(j\) in \(K_t\) tight, if \(\alpha_j = \hat{c}_{i,j}\). Once the pair \((i, j)\) is tight, it increases \(\beta_{i,j}\) at the same rate as \(\alpha_j\) so that \(\alpha_j - \beta_{i,j} = \hat{c}_{i,j}\) is satisfied. The algorithm keeps increasing \(\delta\) until there exists no unfrozen client. As \(\delta\) increases, the following events may occur.
Event 1. If
\[
\sum_{j \in K_t} \beta_{i,j} = \hat{f}_i
\]
for a closed facility \(i\) in \(F\), then \(i\) is temporarily open. In addition, the algorithm freezes unfrozen clients \(j\) in \(K_t\) with \(\beta_{i,j} > 0\) and we call \(i\) the witness for \(j\).

Event 2. If \(\alpha_j = \hat{c}_{i,j}\) for an open/temporarily open facility \(i\) and an unfrozen client \(j\), then the algorithm freezes \(j\) and we call \(i\) the witness for \(j\).

Event 3. If
\[
\sum_{j \in X} \alpha_j = \rho_{K_t}(X)
\]
for a subset \(X\) of \(K_t\), then the algorithm freezes unfrozen clients in \(X\) and adds all elements in \(X\) to \(P\).

If several events occur simultaneously, the algorithm executes them in an arbitrary order.

Next we explain the second phase. Denote by \(T\) the set of temporarily open facilities in \(F\). Facilities \(i, i'\) in \(F\) are said to be dependent, if there exists a client \(j\) in \(K_t\) such that \(\beta_{i,j} > 0\) and \(\beta_{i',j} > 0\). In this phase, we first choose a maximal independent subset \(T'\) of \(T\), and facilities in \(T'\) are open. Then, the algorithm outputs \((Y_t, P_t, \psi_t)\) defined as follows.

- Define \(Y_t\) as the set of open facilities in \(F \setminus H_t\).
- Define \(P_t := P\).
- For each client \(j\) in \(K_t \setminus P_t\), define \(\psi_t(j)\) as an open facility \(i\) in \(F\) minimizing \(\hat{c}_{i,j}\).

In the same as the proof of Lemma 3.1 of [3], we can prove that this algorithm can be implemented in polynomial time. Furthermore, since the pair \((i, j)\) of a client \(j\) in \(K_t\) and a facility \(i\) in \(\sigma_t(j)\) never be tight, \((Y_t, P_t, \psi_t)\) is a feasible solution of \(S(H_t, K_t, \sigma_t)\).

From now on, we analyze an approximation ratio of the algorithm. In the same as Lemma 3.2 of [3], we can prove that during the algorithm’s execution, we have
\[
\sum_{j \in P} \alpha_j = \rho_{K_t}(P).
\]
It follows from this observation that
\[
\alpha_j \ (j \in K_t), \quad \beta_{i,j} \ (i \in H_t \setminus K_t, j \in K_t)
\]
obtained in the first phase is a feasible solution of (11), which implies
\[
\sum_{j \in K_t} \alpha_j \leq \text{OPT}_{\text{SLP}}(t).
\]
We denote by \(F_{\text{op}}\) the set of open facilities in \(F\). For each client \(j\) in \(K_t \setminus P_t\), we denote by \(w(j)\) the witness for \(j\). For each open facility \(j\) in \(F\), we denote by \(N_j\) the set of clients \(j\) in \(K_t\) with \(\beta_{i,j} > 0\). Notice that \(N_i \cap N_{i'}\) is empty for every distinct facilities \(i, i'\) in \(F_{\text{op}}\). Define
\[
D_{\text{po}} := \{j \in K_t \setminus P_t \mid \exists i \in F_{\text{op}}: j \in N_i\}
\]
\[
D_1 := \{j \in K_t \setminus (P_t \cup D_{\text{po}}) \mid w(j) \in F_{\text{op}}\}
\]
\[
D_2 := K_t \setminus (P_t \cup D_{\text{po}} \cup D_1).
\]
Now we prove
\[
\sum_{j \in K_i \setminus P_i} c_{\psi(j),j} = \sum_{j \in K_i \setminus P_i} \hat{c}_{\psi(j),j} \leq \sum_{i \in F_{op}} \sum_{j \in N_i \setminus P_i} \hat{c}_{i,j} + \sum_{j \in D_1} \sum_{j \in D_2} 3 \cdot \alpha_j. \tag{17}
\]

The first inequality follows from the fact that no client \( j \) in \( K_i \) is not connected to facilities in \( \sigma(j) \). For proving the second inequality, we consider the following three cases.

**Case 1.** We first consider the connecting costs for clients in \( D_{po} \). For each client \( j \) in \( D_{po} \), we denote by \( p(j) \) the unique facility \( i \) in \( F_{op} \) with \( j \in N_i \). We have
\[
\sum_{j \in D_{po}} \hat{c}_{\psi(j),j} \leq \sum_{j \in D_{po}} \hat{c}_{p(j),j} \leq \sum_{i \in F_{op}} \sum_{j \in N_i} \hat{c}_{i,j}.
\]

**Case 2.** Next we consider the connecting cost for a client \( j \) in \( D_1 \). Since \( w(j) \) is open and \( j \) is not in \( D_{po} \), we have \( \beta_{w(j),j} = 0 \). This implies that the event 2 occurred when the algorithm froze \( j \), i.e., \( \alpha_j = \hat{c}_{w(j),j} \). Thus, since \( w(j) \) is open, we have
\[
\sum_{j \in D_1} \hat{c}_{\psi(j),j} \leq \sum_{j \in D_1} \hat{c}_{w(j),j} = \sum_{j \in D_1} \alpha_j.
\]

**Case 3.** Here we consider the connecting cost for a client \( j \) in \( D_2 \). Define \( i := w(j) \). In this case, there exist an open facility \( i' \) in \( F \) and a client \( j' \) in \( K_i \) such that \( \beta_{i,j'} > 0 \) and \( \beta_{i',j'} > 0 \). Since \( \beta_{i,j'} \) and \( \beta_{i',j'} \) are positive, \( i \) and \( i' \) are not in \( H_t \). This implies that the triangle inequality holds for \( \hat{c}_{i,j}, \hat{c}_{i',j}, \hat{c}_{i,j'}, \) and \( \hat{c}_{i',j'} \). Let \( t_i \) and \( t_{i'} \) be the times at which \( i \) and \( i' \) are temporarily open, respectively. In addition, the following facts immediately follow.

- Since \( i \) is the witness for \( j \), we have \( \alpha_j \geq t_i \) and \( \alpha_j \geq \hat{c}_{i,j} \).
- Since the pairs \((i, j')\) and \((i', j')\) are tight, we have \( \alpha_{j'} \geq \hat{c}_{i,j'} \) and \( \alpha_{j'} \geq \hat{c}_{i',j'} \).
- Since \( j' \) is frozen earlier than the time \( \min\{t_i, t_{i'}\} \), we have \( \alpha_{j'} \leq \min\{t_i, t_{i'}\} \).

It follows from these facts and the triangle inequality that
\[
\hat{c}_{\psi(j),j} \leq \hat{c}_{i,j} \leq \hat{c}_{i,j'} + \hat{c}_{i',j'} \leq 2\alpha_{j'} + \alpha_j \leq 3\alpha_j.
\]

Hence, we have
\[
\sum_{j \in D_2} \hat{c}_{\psi(j),j} \leq \sum_{j \in D_2} 3 \cdot \alpha_j,
\]
which completes the proof of (17).

Since \( N_i \cap N_{i'} \) is empty for every distinct facilities \( i, i' \) in \( F_{op} \), we have
\[
\sum_{i \in Y_t} = \sum_{i \in F_{op}} \sum_{j \in N_i} \beta_{i,j}
\]
In addition, we have
\[
\sum_{i \in F_{op}} \sum_{j \in N_i} \beta_{i,j} + \sum_{i \in F_{op}} \sum_{j \in N_i \setminus P_i} \hat{c}_{i,j} \leq \sum_{i \in F_{op}} \sum_{j \in N_i} (\beta_{i,j} + \hat{c}_{i,j}) \leq \sum_{j \in D_{po}} \alpha_j + \sum_{j \in P_i} \alpha_j.
\]
It follows from these observations and (17) that
\[
\sum_{i \in Y_1} f_i + \sum_{j \in K_i \setminus P_t} c_{\psi(j), j} + \rho_{K_i}(P_t) \leq \sum_{j \in D_{pp}} \alpha_j + \sum_{j \in D_1} \alpha_j + \sum_{j \in D_2} 3 \cdot \alpha_j + \sum_{j \in P_t} 2 \cdot \alpha_j
\]
\[
\leq 3 \sum_{j \in K_i} \alpha_j
\]
\[
\leq 3 \cdot \text{OPT}_{SLP}(t).
\]

This completes the proof.

5 Conclusion

In this paper, we introduced the fault-tolerant facility location problem with submodular penalties, and presented a combinatorial $3 \cdot H_R$-approximation algorithm, where $R$ is the maximum connectivity requirement. One direction of future work is to improve an approximation ratio. To discern whether we can extend a constant approximation algorithm for the fault-tolerant facility location problem to our problem is interesting. Another direction is to generalize a penalty function. In discrete convex analysis, it is known that the Lovász extensions of submodular functions coincide with polyhedral L-convex functions that are positively homogenous (see [10] for discrete convex analysis). Thus, it is interesting to consider the problem in which the Lovász extension is replaced by a more general discrete convex function.

References


List of MI Preprint Series, Kyushu University
The Global COE Program
Math-for-Industry Education & Research Hub

MI

MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Abstract collision systems simulated by cellular automata

MI2008-2 Eiji ONODERA
The initial value problem for a third-order dispersive flow into compact almost Hermitian manifolds

MI2008-3 Hiroaki KIDO
On isosceles sets in the 4-dimensional Euclidean space

MI2008-4 Hirofumi NOTSU
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme

MI2008-5 Yoshiyasu OZEKI
Torsion points of abelian varieties with values in infinite extensions over a p-adic field

MI2008-6 Yoshiyuki TOMIYAMA
Lifting Galois representations over arbitrary number fields

MI2008-7 Takehiro HIROTSU & Setsuo TANIGUCHI
The random walk model revisited

MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition

MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA
Alpha-determinant cyclic modules and Jacobi polynomials

MI2008-10 Sangyeol LEE & Hiroki MASUDA
Jarque-Bera Normality Test for the Driving Lévy Process of a Discretely Observed Univariate SDE

MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA
A third order dispersive flow for closed curves into almost Hermitian manifolds

MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO
On the $L^2$ a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator

MI2008-13 Jacques FARAUT and Masato WAKAYAMA
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials
MI2008-14 Takashi NAKAMURA
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality

MI2008-15 Takashi NAKAMURA
Some topics related to Hurwitz-Lerch zeta functions

MI2009-1 Yasuhide FUKUMOTO
Global time evolution of viscous vortex rings

MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI
Regularized functional regression modeling for functional response and predictors

MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI
Variable selection for functional regression model via the $L_1$ regularization

MI2009-4 Shuichi KAWANO & Sadanori KONISHI
Nonlinear logistic discrimination via regularized Gaussian basis expansions

MI2009-5 Toshiro HIRANOUCHI & Yuichiro TAGUCHII
Flat modules and Groebner bases over truncated discrete valuation rings

MI2009-6 Kenji KAJIWARA & Yasuhiro OHTA
Bilinearization and Casorati determinant solutions to non-autonomous 1+1 dimensional discrete soliton equations

MI2009-7 Yoshiyuki KAGEI
Asymptotic behavior of solutions of the compressible Navier-Stokes equation around the plane Couette flow

MI2009-8 Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI
Nonlinear regression modeling via the lasso-type regularization

MI2009-9 Takeshi TAKAISHI & Masato KIMURA
Phase field model for mode III crack growth in two dimensional elasticity

MI2009-10 Shingo SAITO
Generalisation of Mack’s formula for claims reserving with arbitrary exponents for the variance assumption

MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA
Ultradiscretization of a solvable two-dimensional chaotic map associated with the Hesse cubic curve

MI2009-12 Tetsu MASUDA
Hypergeometric $\tau$-functions of the q-Painlevé system of type $E_8^{(1)}$

MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic Decomposition for Quantifier Elimination

MI2009-14 Yasunori MAEKAWA
On Gaussian decay estimates of solutions to some linear elliptic equations and its applications
Yuya ISHIHARA & Yoshiyuki KAGEI
Large time behavior of the semigroup on $L^p$ spaces associated with the linearized compressible Navier-Stokes equation in a cylindrical domain

Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE
Spectrum in multi-species asymmetric simple exclusion process on a ring

Masato WAKAYAMA & Keitaro YAMAMOTO
Non-linear algebraic differential equations satisfied by certain family of elliptic functions

Me Me NAING & Yasuhide FUKUMOTO
Local Instability of an Elliptical Flow Subjected to a Coriolis Force

Mitsunori KAYANO & Sadanori KONISHI
Sparse functional principal component analysis via regularized basis expansions and its application

Shuichi KAWANO & Sadanori KONISHI
Semi-supervised logistic discrimination via regularized Gaussian basis expansions

Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations

Eiji ONODERA
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces

Stjepan LUGOMER & Yasuhide FUKUMOTO
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions

Yu KAWAKAMI
Recent progress in value distribution of the hyperbolic Gauss map

Takehiko KINOSHITA & Mitsuhiro T. NAKAO
On very accurate enclosure of the optimal constant in the a priori error estimates for $H^2_0$-projection

Manabu YOSHIDA
Ramification of local fields and Fontaine’s property (Pm)

Yu KAWAKAMI
Value distribution of the hyperbolic Gauss maps for flat fronts in hyperbolic three-space

Masahisa TABATA
Numerical simulation of fluid movement in an hourglass by an energy-stable finite element scheme

Yoshiyuki KAGEI & Yasunori MAEKAWA
Asymptotic behaviors of solutions to evolution equations in the presence of translation and scaling invariance
Yoshiyuki KAGEI & Yasunori MAEKAWA
On asymptotic behaviors of solutions to parabolic systems modelling chemotaxis

Masato WAKAYAMA & Yoshinori YAMASAKI
Hecke’s zeros and higher depth determinants

Olivier PIRONNEAU & Masahisa TABATA
Stability and convergence of a Galerkin-characteristics finite element scheme of lumped mass type

Chikashi ARITA
Queueing process with excluded-volume effect

Kenji KAJIWARA, Nobutaka NAKAZONO & Teruhisa TSUDA
Projective reduction of the discrete Painlevé system of type $(A_2 + A_1)^{(1)}$

Yosuke MIZUYAMA, Takamasa SHINDE, Masahisa TABATA & Daisuke TAGAMI
Finite element computation for scattering problems of micro-hologram using DtN map

Reiichiro KAWAI & Hiroki MASUDA
Exact simulation of finite variation tempered stable Ornstein-Uhlenbeck processes

Hiroki MASUDA
On statistical aspects in calibrating a geometric skewed stable asset price model

Nobuyuki IKEDA & Setsuo TANIGUCHI
The Itô-Nisio theorem, quadratic Wiener functionals, and 1-solitons

Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and detecting change point via the relevance vector machine

Shuichi KAWANO, Toshihiro MISUMI & Sadanori KONISHI
Semi-supervised logistic discrimination via graph-based regularization

Teruhisa TSUDA
UC hierarchy and monodromy preserving deformation

Takahiro ITO
Abstract collision systems on groups
MI2010-9 Hiroshi YOSHIDA, Kinji KIMURA, Naoki YOSHIDA, Junko TANAKA & Yoshihiro MIWA
An algebraic approach to underdetermined experiments

MI2010-10 Kei HIROSE & Sadanori KONISHI
Variable selection via the grouped weighted lasso for factor analysis models

MI2010-11 Katsusuke NABESHIMA & Hiroshi YOSHIDA
Derivation of specific conditions with Comprehensive Groebner Systems

MI2010-12 Yoshiyuki KAGEI, Yu NAGAFUCHI & Takeshi SUDOU
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Poiseuille type flow

MI2010-13 Reiichiro KAWAI & Hiroki MASUDA
On simulation of tempered stable random variates

MI2010-14 Yoshiyasu OZEKI
Non-existence of certain Galois representations with a uniform tame inertia weight

MI2010-15 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of a Rotating Flow Driven by Precession of Arbitrary Frequency

MI2010-16 Yu KAWAKAMI & Daisuke NAKAJO
The value distribution of the Gauss map of improper affine spheres

MI2010-17 Kazunori YASUTAKE
On the classification of rank 2 almost Fano bundles on projective space

MI2010-18 Toshimitsu TAKAESU
Scaling limits for the system of semi-relativistic particles coupled to a scalar bose field

MI2010-19 Reiichiro KAWAI & Hiroki MASUDA
Local asymptotic normality for normal inverse Gaussian Lévy processes with high-frequency sampling

MI2010-20 Yasuhide FUKUMOTO, Makoto HIROTA & Youichi MIE
Lagrangian approach to weakly nonlinear stability of an elliptical flow

MI2010-21 Hiroki MASUDA
Approximate quadratic estimating function for discretely observed Lévy driven SDEs with application to a noise normality test

MI2010-22 Toshimitsu TAKAESU
A Generalized Scaling Limit and its Application to the Semi-Relativistic Particles System Coupled to a Bose Field with Removing Ultraviolet Cutoffs

MI2010-23 Takahiro ITO, Mitsuhiko FUJIO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Composition, union and division of cellular automata on groups

MI2010-24 Toshimitsu TAKAESU
A Hardy’s Uncertainty Principle Lemma in Weak Commutation Relations of Heisenberg-Lie Algebra
MI2010-25  Toshimitsu TAKAESU  
On the Essential Self-Adjointness of Anti-Commutative Operators

MI2010-26  Reiichiro KAWAI & Hiroki MASUDA  
On the local asymptotic behavior of the likelihood function for Meixner Lévy processes under high-frequency sampling

MI2010-27  Chikashi ARITA & Daichi YANAGISAWA  
Exclusive Queueing Process with Discrete Time

MI2010-28  Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA  
Motion and Bäcklund transformations of discrete plane curves

MI2010-29  Takanori YASUDA, Masaya YASUDA, Takeshi SHIMOYAMA & Jun KOGURE  
On the Number of the Pairing-friendly Curves

MI2010-30  Chikashi ARITA & Kohei MOTEGI  
Spin-spin correlation functions of the $q$-VBS state of an integer spin model

MI2010-31  Shohei TATEISHI & Sadanori KONISHI  
Nonlinear regression modeling and spike detection via Gaussian basis expansions

MI2010-32  Nobutaka NAKAZONO  
Hypergeometric $\tau$ functions of the $q$-Painlevé systems of type $(A_2 + A_1)^{(1)}$

MI2010-33  Yoshiyuki KAGEI  
Global existence of solutions to the compressible Navier-Stokes equation around parallel flows

MI2010-34  Nobushige KUROKAWA, Masato WAKAYAMA & Yoshinori YAMASAKI  
Milnor-Selberg zeta functions and zeta regularizations

MI2010-35  Kissani PERERA & Yoshihiro MIZOGUCHI  
Laplacian energy of directed graphs and minimizing maximum outdegree algorithms

MI2010-36  Takanori YASUDA  
CAP representations of inner forms of $Sp(4)$ with respect to Klingen parabolic subgroup

MI2010-37  Chikashi ARITA & Andreas SCHADSCHNEIDER  
Dynamical analysis of the exclusive queueing process

MI2011-1  Yasuhide FUKUMOTO & Alexander B. SAMOKHIN  
Singular electromagnetic modes in an anisotropic medium

MI2011-2  Hiroki KONDO, Shingo SAIITO & Setsuo TANIGUCHI  
Asymptotic tail dependence of the normal copula

MI2011-3  Takehiro HIROTsu, Hiroki KONDO, Shingo SAIITO, Takuya Sato, Tatsushi TANAKA & Setsuo TANIGUCHI  
Anderson-Darling test and the Malliavin calculus

MI2011-4  Hiroshi INOUE, Shohei TATEISHI & Sadanori KONISHI  
Nonlinear regression modeling via Compressed Sensing
MI2011-5 Hiroshi INOUE
Implications in Compressed Sensing and the Restricted Isometry Property

MI2011-6 Daeju KIM & Sadanori KONISHI
Predictive information criterion for nonlinear regression model based on basis expansion methods

MI2011-7 Shohei TATEISHI, Chiaki KINJOY & Sadanori KONISHI
Group variable selection via relevance vector machine

MI2011-8 Jan BREZINA & Yoshiyuki KAGEI
Decay properties of solutions to the linearized compressible Navier-Stokes equation around time-periodic parallel flow
Group variable selection via relevance vector machine

MI2011-9 Chikashi ARITA, Arvind AYYER, Kirone MALLICK & Sylvain PROLHAC
Recursive structures in the multispecies TASEP

MI2011-10 Kazunori YASUTAKE
On projective space bundle with nef normalized tautological line bundle

MI2011-11 Hisashi ANDO, Mike HAY, Kenji KAJIWARA & Tetsu MASUDA
An explicit formula for the discrete power function associated with circle patterns of Schramm type

MI2011-12 Yoshiyuki KAGEI
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a parallel flow

MI2011-13 Vladimír CHALUPECKÝ & Adrian MUNTEAN
Semi-discrete finite difference multiscale scheme for a concrete corrosion model: approximation estimates and convergence

MI2011-14 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Explicit solutions to the semi-discrete modified KdV equation and motion of discrete plane curves

MI2011-15 Hiroshi INOUE
A generalization of restricted isometry property and applications to compressed sensing

MI2011-16 Yu KAWAKAMI
A ramification theorem for the ratio of canonical forms of flat surfaces in hyperbolic three-space

MI2011-17 Naoyuki KAMIYAMA
Matroid intersection with priority constraints

MI2012-1 Kazufumi KIMOTO & Masato WAKAYAMA
Spectrum of non-commutative harmonic oscillators and residual modular forms

MI2012-2 Hiroki MASUDA
Mighty convergence of the Gaussian quasi-likelihood random fields for ergodic Levy driven SDE observed at high frequency
MI2012-3 Hiroshi INOUE
A Weak RIP of theory of compressed sensing and LASSO

MI2012-4 Yasuhide FUKUMOTO & Youich MIE
Hamiltonian bifurcation theory for a rotating flow subject to elliptic straining field

MI2012-5 Yu KAWAKAMI
On the maximal number of exceptional values of Gauss maps for various classes of surfaces

MI2012-6 Marcio GAMEIRO, Yasuaki HIRAOKA, Shunsuke IZUMI, Miroslav KRAMAR, Konstantin MISCHAIKOW & Vidit NANDA
Topological Measurement of Protein Compressibility via Persistence Diagrams

MI2012-7 Nobutaka NAKAZONO & Seiji NISHIOKA
Solutions to a $q$-analog of Painlevé III equation of type $D_7^{(1)}$

MI2012-8 Naoyuki KAMIYAMA
A new approach to the Pareto stable matching problem

MI2012-9 Jan BREZINA & Yoshiyuki KAGEI
Spectral properties of the linearized compressible Navier-Stokes equation around time-periodic parallel flow

MI2012-10 Jan BREZINA
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a time-periodic parallel flow

MI2012-11 Daeju KIM, Shuichi KAWANO & Yoshiyuki NINOMIYA
Adaptive basis expansion via the extended fused lasso

MI2012-12 Masato WAKAYAMA
On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators

MI2012-13 Masatoshi OKITA
On the convergence rates for the compressible Navier- Stokes equations with potential force

MI2013-1 Abduuwali PAERHATI & Yasuhide FUKUMOTO
A Counter-example to Thomson-Tait-Chetayev’s Theorem

MI2013-2 Yasuhide FUKUMOTO & Hirofumi SAKUMA
A unified view of topological invariants of barotropic and baroclinic fluids and their application to formal stability analysis of three-dimensional ideal gas flows

MI2013-3 Hiroki MASUDA
Asymptotics for functionals of self-normalized residuals of discretely observed stochastic processes

MI2013-4 Naoyuki KAMIYAMA
On Counting Output Patterns of Logic Circuits

MI2013-5 Hiroshi INOUE
RIPless Theory for Compressed Sensing
MI2013-6 Hiroshi INOUE
Improved bounds on Restricted isometry for compressed sensing

MI2013-7 Hidetoshi MATSUI
Variable and boundary selection for functional data via multiclass logistic regression modeling

MI2013-8 Hidetoshi MATSUI
Variable selection for varying coefficient models with the sparse regularization

MI2013-9 Naoyuki KAMIYAMA
Packing Arborescences in Acyclic Temporal Networks

MI2013-10 Masato WAKAYAMA
Equivalence between the eigenvalue problem of non-commutative harmonic oscillators and existence of holomorphic solutions of Heun’s differential equations, eigenstates degeneration, and Rabi’s model

MI2013-11 Masatoshi OKITA
Optimal decay rate for strong solutions in critical spaces to the compressible Navier-Stokes equations

MI2013-12 Shuichi KAWANO, Ibuki HOSHINA, Kazuki MATSUDA & Sadanori KONISHI
Predictive model selection criteria for Bayesian lasso

MI2013-13 Hayato CHIBA
The First Painleve Equation on the Weighted Projective Space

MI2013-14 Hidetoshi MATSUI
Variable selection for functional linear models with functional predictors and a functional response

MI2013-15 Naoyuki KAMIYAMA
The Fault-Tolerant Facility Location Problem with Submodular Penalties