A Counter-example to Thomson-Tait-Chetayev’s Theorem

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MI 2013-1

(Received January 8, 2013)
In general, dissipation tends to calm down the motion, and is liable to be regarded as a stabilizing agent. As opposed to this intuition, there are an abundance of phenomena belonging to the category of dissipation-induced instabilities, over a wide range of the fields including mechanical systems, and motion of solids, fluids and plasmas. Dissipation-induced instability originates from a negative-energy mode of a Hamiltonian system which resides only around a non-trivial or moving stationary state.

There is a simple mechanical system that illustrates stabilizing effect by a gyroscopic force but that the stability is lost by an introduction of arbitrary small dissipative force.  

\[ \ddot{q}_1 + \Omega \dot{q}_2 + \delta \dot{q}_1 + c_1 q_1 = 0, \]  
\[ \ddot{q}_2 - \Omega \dot{q}_1 + \delta \dot{q}_2 + c_2 q_2 = 0, \]

where \( q(t) = (q_1(t), q_2(t)) \) are functions of time \( t \), with a dot signifying the differentiation in \( t \), and the terms endowed with constants \( \Omega \) and \( \delta \) (\( \geq 0 \)) represents gyroscopic and dissipative forces, respectively. The last terms represent the potential force linear in \( q(t) \). In the absence of both the gyroscopic (\( \Omega = 0 \)) and the dissipative forces (\( \delta = 0 \)), when at least one of constants \( c_1 \) and \( c_2 \) is negative, the equilibrium state \( q = q_0 = 0 \) of the system (1) and (2) is spectrally unstable. When the both constants \( c_1 \) and \( c_2 \) are negative, this equilibrium state is stabilized by the gyroscopic force if \( |\Omega| > \sqrt{-c_1 + \sqrt{-c_1 c_2}} \). But the stability is lost by addition of the dissipative force however small it is. This result is summarized in the following general statement which is attributed to Thomson and Tait and Chetayev.

**Theorem** If a system without gyroscopic or dissipative forces has a nonzero degree of instability, the equilibrium remains unstable after the addition of gyroscopic and the dissipative forces.

This theorem implies that, if an unstable equilibrium is stabilized by a gyroscopic force, then the stability is destroyed by an introduction of arbitrary dissipative force. An example is brought from the motion of a heavy symmetrical top or a Lagrange top; (1) and (2) constitute the linearized equations around a stationary state of a Lagrange top, for which \( q_1(t) \) and \( q_2(t) \) are two of the Euler angles. The purpose of this letter is to demonstrate that there is an example of a mechanical system close to a Lagrange top which is exempted from Thomson-Tait-Chetayev’s theorem (TTC theorem). We remark that different counter-examples to TTC theorem had been found by a systematic mathematical analysis.

The motion of a Lagrange top is typically described in terms of the Euler angles which specify the configuration of the top as an element of SO(3), the special orthogonal group, in relation to the laboratory frame. The Lie-Poisson equations for the angular momentum and the gravity-vertical axis viewed from the frame fixed to the body provides another means for treatment. Because of the rotational symmetry about the top axis, the equations of motion are reduced only for the top axis by forgetting the rotation of the axis about itself. This amounts to reduce the configuration space SO(3), by the quotient with respect to \( S^1 \), to the spherical surface \( S^2 \cong SO(3)/S^1 \). This process reduces the system on SO(3) to a system governing the orbit of the unit vector \( t(t) \) parallel to the top axis as follows (see also ref\(^{[10]}\)).

Consider the motion of a rigid body with rotational symmetry about an axis, with one point \( O \) on it fixed in space, exerted by the gravity force. This assumption implies that one of the principal axes of the inertia tensor is coincident with the symmetric axis with identical components of the tensor with respect to the axes orthogonal to it and that the center of gravity lies on the symmetric axis. This is the setting of a Lagrange top. If the fixed point \( O \) of the top axis does not coincide with the center of gravity, as a generic case, the gravity force exerts a torque on the top about \( O \).

Let \( \hat{t}(t) \) be the unit vector along the axis of symmetry, and \( e_1(t) \) and \( e_2(t) \) be unit orthogonal vectors in the plane perpendicular to it, fixed to the top, as functions of the time \( t \). If we denote the angular velocity of the body by \( \omega(t) \), the motion of the top axis is written as

\[ \dot{t} = \omega \times t. \]
Taking the vector product of \( t \) with (4), we have
\[
\omega = t \times t + \omega_3 t, \tag{4}
\]
where \( \omega_3 = \omega \cdot t \) is the axial component of the angular velocity, which is shown to be a constant of the motion associated with the rotational symmetry of the body. The angular momentum \( M(t) \) relative to the stationary point \( O \), viewed from the inertial frame, is coined by imparting the moments of inertia about the stationary point \( O \) to the corresponding components of the angular velocity, giving,
\[
M = A t \times i + C \omega_3 t, \tag{5}
\]
where \( C \) and \( A \) are the moments of inertia about \( O \) with respect to the axial direction and the directions perpendicular to it, respectively. Newton’s second laws of mechanics states that the time-wise rate of change of the angular momentum about the stationary point \( O \) is equal to the moment of force about \( O \). In this letter, we apply, in addition to the gravity force, the drag force, proportional to the speed \( t \) of the axis, acting on the top axis in the direction of exactly opposing the motion. Without loss of generality, we may take the drag force \(-\delta t\) to act on the center of gravity, where \(-\delta (\geq 0)\) is a small positive constant. Newton’s law dictates
\[
M = lt \times (-mge.) + lt \times (-\delta t), \tag{6}
\]
where \( m \) is the mass of the body, \(-g e.\) is the gravity acceleration with \( e.\) being the unit vector in the \( z\)-direction taken vertically upwards, and \( l \) is the length of line segment connecting the stationary point \( O \) to the centre of mass. The last term stems from the drag force. We emphasize that this drag or friction is different from the usual frictional force acted at \( O \) by the boundary surface to slow down the rotation speed of the top axis, and thus to decrease \( |\omega_3| \). In our model (6), \( \omega_3 \) is constant.

This friction term acquires reality if we translate (6) into the equations for a spherical pendulum. Substituting (5) into (6) and thereafter taking the vector product with \( t \), we are left, after rearranging the terms, with
\[
A \ddot{\delta} = -mg[l e_2 - (t \cdot e_2)t] - A(t \cdot i) t - C \omega_3 i \times t - \delta t, \tag{7}
\]
where \( \delta = \dot{\delta} (\geq 0) \). This system of equations are interpreted as representing the motion of a spherical pendulum with the gravity acceleration replaced by \( mgl/A \), being directed in the negative \( z\)-axis. The first and second terms on the right-hand side of Eq. (7) signify that a point mass is exerted by the gravity force, the normal component of which is being projected out. The third one is the centrifugal force, and the fourth one is the Lorentz force in the field of a magnetic monopole located at \( O \). In accordance, the point mass is endowed with charge, and the ratio of the monopole field to the charge is \(-C \omega_3 / A \).\(^{30}\) The same force as this Lorentz force can be generated by the Magnus force, of aerodynamic origin, where the pendulum, of finite size, is rotating about \( t \). The last term is the drag force. In the context of a spherical pendulum, our drag force looks more natural.

When the drag force is switched off \( (\delta = 0) \), (6) and (7) are equivalent to the equations governing a Lagrange top. However, some difference makes its appearance when we make headway to the stability analysis. Our system (7) describes an orbit on \( S^2 \), the reduced space. They describe only the time-wise change of the orientation of the axis \( t(t) \) with maintaining the value of \( \omega_3 \). The rotation angle about \( t(t) \) is prescribed in the sense that the perturbation does not affect \( \omega_3 \), though this is not the case with a Lagrange top.

For our purpose of making the stability analysis of stationary states, we rewrite the vector equation (7) in terms of a spherical coordinate system \((\theta, \phi)\) defined via \( t = (\sin \theta \cos \phi, \cos \theta \sin \phi, \cos \theta) \). By projecting (7) to \( e_\theta = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta) \) and to \( e_\phi = (-\sin \phi, \cos \phi, 0) \), we obtain, respectively,
\[
A(\dot{\theta} - \delta \sin \theta \cos \phi = mg l \sin \theta - C \omega_3 \phi \sin \theta - \delta \theta, \tag{8}
\]
\[
A(\ddot{\phi} \sin \theta + 2\dot{\phi} \cos \theta = C \omega_3 \theta - \delta \phi \sin \theta. \tag{9}
\]
Observe that our coupled equations (8) and (9) bear some resemblance with the prototypical linear gyroscopic system (1) and (2) augmented with the dissipative force. There are gyroscopic terms, with coefficient \( C \omega_3 \), on the right-hand side of (8) and (9). The gyroscopic force is generated by rotation \((\omega_3 \neq 0)\) of the top axis about itself, and its strength is proportional to the axial component \( M_3 = C \omega_3 \) of the angular momentum. The last terms, having the coefficient \( \delta \), signify the drag force. A marked difference lie is the fact that coupled equations (8) and (9) are nonlinear in dependent variables \( \theta \) and \( \phi \) in contrast with (1) and (2). In the sequel, we explore how the drag force modifies the spectral stability of typical stationary sates of the heavy symmetrical top.

We start with the stability of the so-called ‘sleeping top’. This is a static state, executing neither nutation nor precession, with the top axis directed vertically upward. This upright orientation is expressed by
\[
\theta(t) \equiv 0 \text{ for all } t, \quad \phi(t) = \Phi(t), \tag{10}
\]
where \( \Phi(0) \) is an arbitrary function of time \( t \). The arbitrary function is admitted because the azimuthal angle \( \phi \) is undefined when \( \sin \theta = 0 \), namely, when \( \theta = 0 \) and \( \pi \). We confirm by direct substitution that (10) is indeed a solution of (8) and (9).

We then consider the linear stability of this upright position (10). Substituting \( \theta(t) = \tilde{\theta}(t) \) and \( \phi(t) = \Phi + \tilde{\phi}(t) \) into (8) and (9) and linearizing in the perturbation amplitude \( |\tilde{\theta}| \) and \( |\tilde{\phi}| \) which are assumed to be of infinitesimal, we are left with
\[
A \ddot{\tilde{\theta}} + (C \omega_3 \Phi - A \tilde{\phi}^2 - mg l \hat{\theta} - \delta \hat{\theta}, \tag{11}
\]
\[
A \left( \ddot{\tilde{\phi}} + 2 \dot{\tilde{\phi}} \dot{\phi} - C \omega_3 \dot{\theta} - \delta \phi \dot{\theta}. \tag{12}
\]
Note that \( \delta \) does not appear. The case of \( \sin \theta = 0 \) is degenerate, in which two equations (11) and (12) are available for determining the single perturbation function \( \tilde{\theta}(t) \). A sensible interpretation could be that the problem of being overdetermined is resolved by compensating for it with arbitrariness of the basic state \( \Phi = \Phi(t) \). Crudely speaking, the time-evolution of \( \tilde{\theta}(t) \) is mainly determined by the first equation (11), and then the disposable function \( \Phi(t) \) is adjusted so as to satisfy the second equation (12).

For the moment, we ignore the drag force \((\delta = 0)\). Since (11) is a second-order linear homogeneous ordinary differential equation for \( \tilde{\theta} \), with a time-dependent coefficient. In such a case, a normal-mode solution of the form \( \tilde{\theta} = e^{\lambda t} \), with constant exponent \( \lambda \) is not allowed. The criterion for the linear stability is provided by the condition that the term proportional to \( \lambda \) is a ‘restoring force’, meaning that its coefficient


\( CW_{3} \Phi - A \dot{\Phi}^2 - mgl > 0 \). It is noteworthy that, if it were not for the gyroscopic force \( CW_{3} = 0 \), \( CW_{3} \Phi - A \dot{\Phi}^2 - mgl < 0 \) and no restoring force is available, being an immediately acceptable result. The Lorentz force generated by the magnetic monopole is requisite for the linear stability. Observe that, when \( CW_{3} \neq 0 \) and the linear perturbation \( \delta(t) \) behaves non-trivially, the second equation does not admit \( \Phi(t) \equiv 0 \). We may assume without loss of generality that \( \omega_{3} > 0 \), and correspondingly assume that \( \delta > 0 \), which is plausible in view of the balance of the second and the third terms on the left-hand side of (12). Then a sufficient condition for the linear stability is

\[
CW_{3} > A \Phi + \frac{mgl}{\Phi} \geq 2 \sqrt{Amgl},
\]

(13)

where use has been made of the relation of the arithmetic mean being greater than the geometric mean. Thus the well-known criterion for the stability of the sleeping top is restored;\(^{11}\) the sleeping top is stabilized if it rotates faster than the critical angular velocity as given above. When the stability condition (13) is met, the \( \dot{\theta} \)-term in (11) is restoring whatever the value of \( \Phi \), of the same sign as \( \omega_{3} \), is taken.

Next, we look into the influence of the drag force \( (\delta > 0) \). By a change of the dependent variable

\[
\hat{\varphi} = \exp \left( -\frac{\delta}{2A} t \right) \varphi,
\]

(14)

the term with \( \delta \) is eliminated from (11), leaving

\[
\ddot{\varphi} + \left( CW_{3} \Phi - A \dot{\Phi}^2 - mgl - \frac{\delta^2}{4A} \right) \hat{\varphi} = 0.
\]

(15)

Since our concern is in the TTC theorem, we restrict our attention to influence of the small drag force on the gyroscopically stable case (13). To linear in \( \delta \), the condition (13) for restoring linear term remains intact

\[
CW_{3} > 2 \sqrt{Amgl} + \frac{\delta^2}{4} \geq 2 \sqrt{Amgl},
\]

(16)

though modification arises at \( O(\delta^2) \). If the gyroscopic force is strong, compared with the gravity and the centrifugal forces, enough for the inequality to be met, the upright orientation \( (\theta = 0) \) is gyroscopically stable. A combination of (14) and (15) indicates that the friction exclusively acts to dam perturbations from the upright state exponentially in time. This result is not consistent with the aspect of the TTC theorem that stability of a state stabilized gyroscopically is lost by an introduction of arbitrary small dissipative force. The degeneracy of (12) at \( \theta = 0 \) prohibits manifestation of the perturbation variable \( \Phi \) from the linearized equations, whence the equilibrium \( \theta = 0 \) slips through the prescription of the TTC based on the linear gyroscopic system (1) and (2). As before, we can exploit the freedom of an arbitrary function \( \Phi(t) \) to fulfill (12). The arbitrariness \( \Phi(t) \) of the basic state of \( \phi \) accords with the disappearance with the perturbation \( \Phi \). With this, we close the description of our main result.

In order to see the relevance of our problem with the dissipation-induced stability, we turn to the precessing motion. This is a steadily rotating motion of a rigid body for which the top axis \( t \) describes a circular cone, with constant angle \( \theta_{0} \) \( (0 < \theta_{0} < \pi) \) between \( t \) and \( e_{3} \), and with constant angular velocity \( \omega_{3} \) of rotation of \( t \) about \( e_{3} \). When the friction terms are discarded, (8) and (9) enforces

\[
\sin \theta_{0} \left( a_{3} \omega_{3} \cos \theta_{0} + ml - CW_{3} \Omega_{3} \right) = 0.
\]

(17)

Since \( \sin \theta_{0} \neq 0 \), (17) determines the angular velocity \( \Omega_{3} \) of the top axis about \( e_{3} \), in relation to the angular velocity \( \omega_{3} \) of the top axis about itself as

\[
\dot{\omega}_{3} \Omega_{3} = \hat{\varphi} + \Omega_{3} \cos \theta_{0},
\]

(18)

where \( \hat{\varphi} = mgl/A \) and \( \omega_{3} = CW_{3}/A \). Note that the sleeping top \((\theta = 0)\) and the gravitational equilibrium \((\theta = \pi)\) are isolated branches from the precessing motion \((\sin \theta_{0} \neq 0)\); they do not stand as the limiting solutions of (18).

We proceed to the linear stability analysis of the precessing motion. Posing the perturbed solution as \( \theta(t) = \theta_{0} + \tilde{\theta}(t) \) and \( \varphi(t) = \Omega_{3} t + \tilde{\varphi}(t) \), and substituting these into (8) and (9), we obtain, after linearization,

\[
\begin{align*}
\ddot{\varphi} & - 2\Omega_{3} \sin \theta_{0} \cos \theta_{0} \dot{\varphi} - \Omega_{3}^{2} (\cos^{2} \theta_{0} - \sin^{2} \theta_{0}) \dot{\theta}\sin \theta_{0} \\
& - \hat{\varphi} \cos \theta_{0} + \omega_{3} \Omega_{3} \cos \theta_{0} + \omega_{3} \dot{\varphi} \sin \theta_{0} \\
& = - \delta \dot{\varphi},
\end{align*}
\]

(19)

\[
\begin{align*}
\ddot{\theta} & - \omega_{3} \Omega_{3} \cos \theta_{0} - \Omega_{3} \dot{\varphi} \sin \theta_{0} \\
& = - \delta \Omega_{3} (\hat{\varphi} \cos \theta_{0} + \dot{\varphi} \sin \theta_{0}) - \delta \Omega_{3} \sin \theta_{0}.
\end{align*}
\]

(20)

As a preliminary step, we seek the spectral stability of frictionless motion \((\delta = 0)\). Substitution of the normal-mode form solution \( \varphi \propto e^{\lambda t}, \varphi \propto e^{\mu t} \) into (19) and (20) leads to the eigenvalue equation

\[
\sin \theta_{0} \lambda^{2} \left[ \omega_{3}^{2} \sin^{2} \theta_{0} + (\omega_{3}^{2} - 2 \Omega_{3} \cos \theta_{0}) \right] = 0,
\]

(21)

Because of \( \sin \theta_{0} \neq 0 \), (21) yields \( \lambda = 0 \) as a degenerate eigenvalue and

\[
\lambda^{2} = - \omega_{3}^{2} \sin^{2} \theta_{0} - (\omega_{3}^{2} - 2 \Omega_{3} \cos \theta_{0}) \equiv -\alpha^{2},
\]

(22)

where \( \alpha > 0 \) is taken. Since \( \alpha^{2} > 0 \), the eigenvalues \( \lambda = \pm i \alpha \) given by (22) are pure imaginary, implying oscillations in time. As regards the spectral stability, the precessing motion is marginally stable with double zero eigenvalues.

We are now in a position to incorporate the influence of the drag force \((\delta \neq 0)\). The coupled equations (19) and (20) represent a forced oscillation with the last term of (20) acting as a forcing term. In this respect, the sleeping top and the gravitational equilibrium \((\sin \theta_{0} = 0)\) are exceptional in the sense that the forcing term disappears. First we inquire into how the drag force affects the spectra by ignoring the forcing term. Repeating the same procedure as above, we find that (21) gives way to

\[
\sin \theta_{0} \lambda \left[ \lambda^{3} + 2 \lambda \omega_{3}^{2} + (\alpha^{2} + \delta^{2}) \lambda + \delta (\omega_{3}^{2} - \hat{\varphi} \cos \theta_{0}) \right] = 0,
\]

(23)

where use has been made of (18). For precessing motion \((\sin \theta_{0} \neq 0, \lambda = 0)\) is a still one of the eigenvalues. The other three go through modification by the friction effect. For weak drag force, the approximate values of the eigenvalues are, to \( O(\delta) \),

\[
\lambda = \left\{ \begin{array}{ll}
\frac{1}{\alpha^{2}} (\hat{\varphi} \cos \theta_{0} - \Omega_{3} \delta) + O(\delta^{2}), \\
\pm i \alpha \left[ -1 + \frac{1}{2 \alpha^{2}} \omega_{3}^{2} (\hat{\varphi} \cos \theta_{0}) \right] + O(\delta^{2}).
\end{array} \right.
\]

(24)

When the real part of at least one of the eigenvalues (24) is
positive, the precessing motion with \( \theta = \theta_0 \) and \( \phi = \Omega_c \) is spectrally unstable, of dissipative origin.

It is well known that a simple spherical pendulum acted only by the gravity force \( (C \omega_3 = 0) \) cannot maintain the pendulum above the fixed point for all time \( t \).\(^{11)} \) In keeping with the theme of a gyroscopically stabilized state, we restrict our attention to \( 0 < \theta_0 < \pi/2 \) or \( 0 < \cos \theta_0 < 1 \) for which the top axis is lifted above the horizontal direction, but not strictly aligned with the vertical axis. The tilting angle \( \theta_0 \) should be initially prescribed. The quantity \( C \omega_3 \) naturally serves as a control parameter. This may be considered to be the angular velocity of the top axis driven by one’s fingers in the context of a Lagrange top or alternatively to be the strength of the magnetic monopole in the context of a charged spherical pendulum. Given the values of \( \hat{g}, \theta_0 \) and \( \omega_3 \), (18) has two roots for \( \Omega_c \), as

\[
\Omega_c = \frac{1}{2} \left( \frac{\dot{\omega}_3}{\cos \theta_0} \pm \sqrt{\frac{\dot{\omega}_3^2}{\cos^2 \theta_0} - \frac{4\hat{g}}{\cos \theta_0}} \right). \tag{25}
\]

In view of the series form (24) of the eigenvalue, the real part of \( \hat{g} \), \( \theta_0 \) and \( \omega_3 \), (18) has two roots for \( \Omega_c \), as

\[
\Omega_c = \frac{1}{2} \left( \frac{\dot{\omega}_3}{\cos \theta_0} \pm \sqrt{\frac{\dot{\omega}_3^2}{\cos^2 \theta_0} - \frac{4\hat{g}}{\cos \theta_0}} \right). \tag{25}
\]

In view of the series form (24) of the eigenvalue, the real part depends on \( \Omega_c^2 \), and hence we may take \( \dot{\omega}_3 \geq 0 \). The motion with the larger \( \Omega_c \) of plus sign, \( \Omega_c^+ \) say, is referred to as the fast precession, and that with the smaller \( \Omega_c \), of minus sign, \( \Omega_c^- \) is referred to as the slow precession.\(^{11)} \) Numerical examples suggest that, for the fast precession, the real parts of the three roots (23) are negative, but that, for the slow precession, the real part of the real root among (23) is positive if \( \omega_3 \) is not very small. The latter case complies with the typical scenario of the dissipation-induced instability. The smaller one \( \Omega_c^- \) is a decreasing function in \( \dot{\omega}_3/\cos \theta_0(> 0) \), and \( \Omega_c^+ \) becomes smaller than \( \hat{g} \cos \theta_0 \) if \( \dot{\omega}_3 > \sqrt{\hat{g}/\cos \theta_0(\cos^2 \theta_0 + 1)} \). Over this wide parameter range, the real root represented in series form in \( \delta \), among (24), \( A_1 \approx (\hat{g} \cos \theta_0 - \Omega_c^2)\delta/\alpha^2 > 0 \), and the dissipation-induced instability is invited.

In this letter, we have highlighted the influence of the drag force on the spectra. However, the drag force supplies the inhomogeneous term as given by the last one of (20). To grasp a crude picture of the overall evolution of the top axis, we draw in Fig. 1 the orbit of the top of \( t \) on the unit sphere by numerically integrating (19) and (20) with a choice of typical parameter values corresponding to a precession. Here we fix \( \hat{g} = 1/2, \dot{\omega}_3 = 2.5, \delta = 0.05 \), and set, as the initial conditions, \( \theta(0) = \pi/6, \dot{\theta}(0) = 0 \) and \( \phi(0) = 0 \), and \( \phi(0) = \Omega_c^- \approx 0.2162 \) (left), the smaller one of (25), and \( \phi(0) = \Omega_c^+ \approx 2.671 \) (right), the larger one. The branch of \( \phi(0) = \Omega_c^- \) suffers from the dissipation-induced instability with the eigenvalues provided by \( \lambda = 0, \lambda \approx 0.004261 \) and \(-0.05213 \pm 2.128i \). For the slow precession, the drag force acts to tilt down the top axis monotonically toward the gravitational equilibrium \( (\theta_0 = 0) \). The fast branch of \( \phi(0) = \Omega_c^+ \) has the eigenvalues, \( \lambda = 0, \lambda \approx -0.005316 \) and \(-0.02342 \pm 2.510i \), being free from the instability. To our surprise, for the fast precession, with no instability, the drag acts to tilt up the top axis toward the upright orientation \( (\theta_0 = 0) \). Notwithstanding the system is losing energy as a whole, the dissipation acts to lift the center of gravity against the gravity force.

The upright orientation of the top axis is a gyroscopically stable equilibrium, which is totally isolated from the precessing motion. This is by no means the limit of the precession, and the equations determining the spectrum are highly degenerate. For small values of the drag force, no dissipation-induced instability is admitted. There are several questions to wait for answers, such as the nonlinear evolution of the sleeping top and the origin of drag-induced lifting up of the axis for the fast precession. These and other issues are left for a future investigation.

Acknowledgment Y. F. is grateful to Peter Lancaster who brought Y. F.’s attention to his inspiring discovery of counter-examples to the TTC theorem.\(^{6)} \) Y. F. was supported in part by a Grant-in-Aid for Scientific Research from the Japan Society for the Promotion of Science (Grant No. 24540407).

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