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The Pattern Formation Problem for Autonomous Mobile Robots

— Special Lecture in Functional Mathematics —

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Preface

Given today's technological advances and decreasing manufacturing cost, mobile robots are expected to be utilized in the near future in a wide range of applications to assist humans, in places like factories, hospitals, museums and households. Since roughly speaking a robot, just like a computer, is hardware (wheels, motors, manipulators, sensors, etc.) controlled by software, a useful robot must have hardware capable of performing the required physical actions *and* software capable of analyzing the sensor output and determining a good course of action for achieving the given goal efficiently in a given (possibly unknown) environment. Consequently, there are two general aspects of robotics research, one on hardware and one on software. In this note we focus on the latter, and give a brief overview of a research topic in mobile robot algorithm design that has received considerable attention in recent years — formation of geometric patterns by autonomous mobile robots under distributed control.

Motion coordination and self-organization of mobile robots under distributed control have been a common research area in automatic control, robotics and computer science. Engineering motivations for investigating self-organization arise in designing algorithms for distributed systems such as mobile sensor networks and autonomous robot teams, where mobile agents are deployed to perform search, monitoring, exploration, etc., adaptively in an unknown environment.

To motivate the pattern formation problem, consider the following scenario. Suppose that a school teacher wants her 100 children in the playground to form a circle. She could use "centralized control" by drawing a circle on the ground as a guideline or even giving each child a specific position to move to. Alternatively, she could use "distributed control" and obtain a fairly good approximation of a circle by asking each child to move adaptively, based on the movement of other children and his/her knowledge of the shape of a circle. We wish to solve such pattern formation problems for autonomous mobile robots, using the latter distributed approach. To simplify the discussion, we model each robot as a mobile automaton that repeats a "Look-Compute-Move" cycle in discrete time steps indefinitely. The robots are anonymous (i.e., they have no ID's and use the same algorithm), and may even be oblivious (i.e., they have no memory to remember the past). They do not have a common x-y coordinate system. Given a group of n such robots and a target geometric pattern F, does there exist an algorithm (to be executed by the robots individually) that guarantees that the robots eventually form F in a finite number of steps regardless of their initial configuration? Work on this topic was initiated by Sugihara and Suzuki in 1990, followed by a first formal study in 1999 by Suzuki and Yamashita. We give an overview of some of the major results, including a recent one. For the benefit of the reader who wishes to study the subject further, a recent survey article by Y. Katayama and M. Yamashita is included in the appendix.

There are strong links between the area of robot algorithm design and some branches of mathematics and computer science, in particular computational geometry and distributed computing. Though it is perhaps true that a robot algorithm must ultimately be tested on a physical system, we hope that the reader of this article, presumably a mathematics student, will be convinced that through abstraction and formal reasoning, he/she can potentially contribute to the utility of robots in the future.

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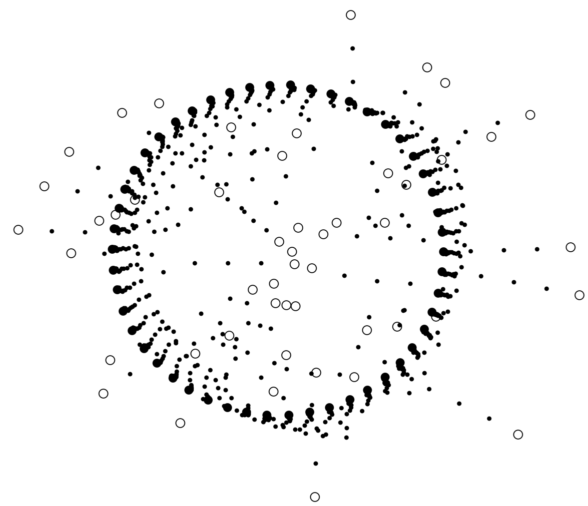

Figure 1: Hollow circles are the initial positions of 50 robots. Solid circles are their final positions after the execution of Tanaka's algorithm. Small dots represent their intermediate positions.

1 Introduction

The problem of forming an approximation of a circle having a given radius R by identical mobile robots was first discussed by Sugihara and Suzuki [15] (see also [16]). Assuming that the positions of the robots are the only information available, they proposed a simple heuristic distributed algorithm for circle formation (to be executed independently by all robots) that, according to simulation results, sometimes brings the robots to a pattern reminiscent of a Reuleaux's triangle rather than a circle. Tanaka [18] later proposed a different algorithm and demonstrated, using simulation, that his algorithm does not have this shortcoming and generates a better approximation of a circle. In essence, in his algorithm each robot regards the midpoint p of the positions of its nearest and furthest neighbors as the center of the circle to be formed, and adjusts its position so that it will be approximately at distance R from p, and if this condition is already satisfied, then it moves away from its nearest neighbor. Fig. 1 shows the behavior of 50 robots executing his algorithm starting from an initial distribution generated randomly. This extremely simple algorithm demonstrates the potential of the distributed method for pattern formation.

The pattern formation problem for anonymous mobile robots has gained much attention in recent years [2] [3] [12] [13] [14]. The problem was discussed formally, together with related convergence and agreement problems, for the first time in [17] for both oblivious robots and non-oblivious robots, in what we call the semi-synchronous (SSYNCH or Suzuki-Yamashita) model and the fully-synchronous (FSYNCH) model. Briefly, each robot r, represented by a point, repeats an instantaneous "Look-Compute-Move" cycle, where in the Look phase it observes the locations of all robots, in the Compute phase it computes its next location, and in the Move phase it moves to that location. All observations and computations of r are done in terms of r's own local coordinate system. (The local coordinate systems of two robots may not agree.) All robots always execute the cycles simultaneously in the FSYNCH model, while in the SSYNCH model, that is not necessarily the case. An oblivious robot is one without memory to remember what it has observed in the past, and hence whose action at any given cycle depends only on what it sees in that cycle. A non-oblivious robot has memory to store what it observes, and hence its action can depend not only on what it currently observes, but also on what it has observed in the past. The robots are anonymous, without identifiers and all executing the same algorithm.

Our goal is to understand the power and limitations of the distributed approach for pattern formation, using the abstract model outlined above. Let us first formalize the model and the problem.

2 The formal model

Let $\mathcal{R} = \{r_1, r_2, \ldots, r_n\}$ be a set of $n \geq 2$ robots situated in a two dimensional space $\mathbf{R}^2$ having a global x-y coordinate system Z, where $\mathbf{R}$ is the set of real numbers. The robots have no identifiers, and the subscript "i" of r_i is used only for convenience of explanation. The robots do not have access to Z, and for each $1 \leq i \leq n$, r_i observes and computes positions only in its local coordinate system Z_i. We assume that Z and Z_i's are all right-hand systems, and specify Z_i by a pair (o_i, d_i), where o_i and d_i ($o_i \neq d_i$), respectively, denote the coordinates (in Z) of the origin $(0, 0)$ and $(1, 0)$ of Z_i. Robot r_i knows Z_i, but it does not know (o_i, d_i) as coordinates in Z. For any point p (given in Z), $[p]_{Z_i}$ denotes the coordinates of p in Z_i. Thus for instance, $[o_i]_{Z_i} = (0, 0)$ and $[d_i]_{Z_i} = (1, 0)$.

We assume discrete time $0, 1, 2, \ldots$, and denote by $p_i(t)$ the position of r_i at time t. We assume that the robots' initial positions $p_1(0), p_1(0), \ldots,\ p_n(0)$ are all distinct. Each robot is modeled as a point, and we allow two or more robots to occupy the same position simultaneously, creating a *multiplicity*. We assume that the robots can detect multiplicities and their sizes (i.e., the number of robots located there). We denote by $P(t) = \{p_1(t), p_2(t), \ldots, p_n(t)\}$ the multiset of positions of the robots at time t, and let

$$[P(t)]_{Z_i} = \{[p_1(t)]_{Z_i}, [p_2(t)]_{Z_i} \ldots, [p_{i-1}(t)]_{Z_i}, [p_i(t)]^*_{Z_i}, [p_{i+1}(t)]_{Z_i}, \ldots, [p_n(t)]_{Z_i}\},$$

be the *sight* of r_i (i.e., what r_i observes in Z_i) at t, where the symbol "$*$" for $[p_i(t)]_{Z_i}$ signifies that r_i is aware of its own position. Intuitively, if $[P(t)]_{Z_i} = [P(t)]_{Z_j}$, then $P(t)$ looks identical to r_i and r_j in their respective local coordinate systems, from their respective positions.

At each time $t \geq 0$, every robot is either *active* or *inactive*. We use A_t to denote the set of active robots at t, and call the sequence $\mathcal{A} = A_0, A_1, \ldots$ an *activation schedule*. We assume that every robot becomes active infinitely many times. An inactive robot does not move; i.e., $p_i(t+1) = p_i(t)$ if $r_i \notin A_t$. An active robot $r_i \in A_t$ executes the following Look-Compute-Move cycle instantaneously as an atomic action: (1) obtain its sight $[P(t)]_{Z_i}$, (2) compute its next position using a given function ψ, and (3) move to that position. Function ψ takes as input the sequence of sights that r_i has obtained so far, including the current sight $[P(t)]_{Z_i}$. Specifically, if $0 \leq t_1 < t_2 < \cdots < t_m = t$ are the times when r_i has been active, and if $q = \psi([P(t_1)]_{Z_i}, [P(t_2)]_{Z_i}, \ldots, [P(t_m)]_{Z_i})$, then $p_i(t+1) = q'$, where q' is the point in Z such that $[q']_{Z_i} = q$. That is, r_i moves to point q of Z_i, which is q' in Z. To impose an upper bound on the "speed" of a robot, we require the distance between r_i's current location $p_i(t_m)$ and q' to be at most the unit distance 1 of Z_i. (Equivalently, q must be within distance 1 of $[p_i(t_m)]_{Z_i}$ in Z_i.) We assume that all active robots reach their intended destinations without colliding with other robots.

In short, robot r_i observes the robot positions as a sight only when it is active, and its next position depends only on ψ and the sights that it has obtained so far, where all observations and computations are done in Z_i. Function ψ is said to be *oblivious* if $\psi([P(t_1)]_{Z_i}, [P(t_2)]_{Z_i}, \ldots, [P(t_m)]_{Z_i}) = \psi([P(t_m)]_{Z_i})$ always holds, i.e., the move of a robot depends only on its current sight. A robot that uses an oblivious function is said to be oblivious.

Note that the robots are *anonymous* in the following sense: (1) function ψ is common to all robots, (2) the identifier "i" of robot r_i is not an argument of ψ, and (3) $[P(t)]_{Z_i}$ contains only the positions of the robots (but not their identities).

The model of the robots described above is called the *semi-synchronous* (SSYNCH or Suzuki-Yamashita) model [17]. If every robot appears in A_t for every t, then it is called the *fully-synchronous* (FSYNCH) model.

3 The problem

Let F be a multiset of n points in Z that describes a target formation (or *pattern*) of the n robots in $\mathcal{R}$. Note that whether the robots can form F using a given function ψ depends not only on their current positions but also on their local coordinate systems (because each r_i operates in Z_i). We represent robot positions p_i of r_i, $1 \leq i \leq n$, together with their local coordinate systems as a *configuration* $I = \{(p_i, Z_i) | 1 \leq i \leq n\}$, and use notation $P(I) = \{p_i | 1 \leq i \leq n\}$ to refer to the robot positions in I. In particular, $I(t) = \{(p_i(t), Z_i) | 1 \leq i \leq n\}$ and $P(I(t)) = P(t)$, respectively, are the configuration and robot positions at time t. Note that a robot can observe $P(t)$ in its own local coordinate system, but not $I(t)$ since the robots' local coordinate systems are "not visible."

A function ψ is said to solve the *formation problem* for pattern F starting from initial configuration $I(0)$, if for *any* activation schedule $\mathcal{A}$, there exists a time instant t such that, for all $t' \geq t$, $P(t')$ is similar to F in the sense that $P(t')$ matches F after suitable transformation, rotation and uniform scaling. If such ψ exists, we say that the robots can form pattern F starting from $I(0)$. In contrast, ψ solves the *convergence problem* for F and $I(0)$ if for *any* activation schedule $\mathcal{A}$, the resulting sequence $P(0), P(1), \ldots$ of robot positions converges to a multiset P that is similar to F. If such ψ exists, we say that the robots can converge to pattern F starting from $I(0)$.

4 Preliminaries

Whether a given pattern can be formed in the above sense depends critically on the degree of symmetry that exists in the robots' initial configuration $I(0)$. It is convenient to define symmetry using rotation.

Let $I = \{(p_i, Z_i) | 1 \leq i \leq n\}$ be a configuration, where $Z_i = (o_i, d_i)$ for each i. Let $\gamma_{o,\theta}$ be a function (rotation) that rotates a given point in $\mathbf{R}^2$ counterclockwise around a point $o \in \mathbf{R}^2$ by an angle θ. We extend $\gamma_{o,\theta}$ by $\gamma_{o,\theta}((p_i, Z_i)) = (\gamma_{o,\theta}(p_i), (\gamma_{o,\theta}(o_i), \gamma_{o,\theta}(d_i)))$, and then extend it again to a rotation of I (around o by θ) by $\gamma_{o,\theta}(I) = \{\gamma_{o,\theta}((p_i, Z_i)) | 1 \leq i \leq n\}$. Now, let $\Gamma_o(I)$ be the set of rotations $\gamma_{o,\theta}$ around o, $0 \leq \theta < 2\pi$, such that $\gamma_{o,\theta}(I) = I$. Then $\Gamma_o(I)$ forms a cyclic group under composition, and we denote its order by $\sigma_o(I) = |\Gamma_o(I)|$. Clearly, $\sigma_o(I)$ is a divisor of n for any o.

Example 1 Let $p_1 = (0,0), p_2 = (1,0), p_3 = (1,1)$ and $p_4 = (0,1)$, and $o_i = p_i$ ($i = 1,2,3,4$). Let $o = (1/2, 1/2)$. See Fig. 2 for illustration.

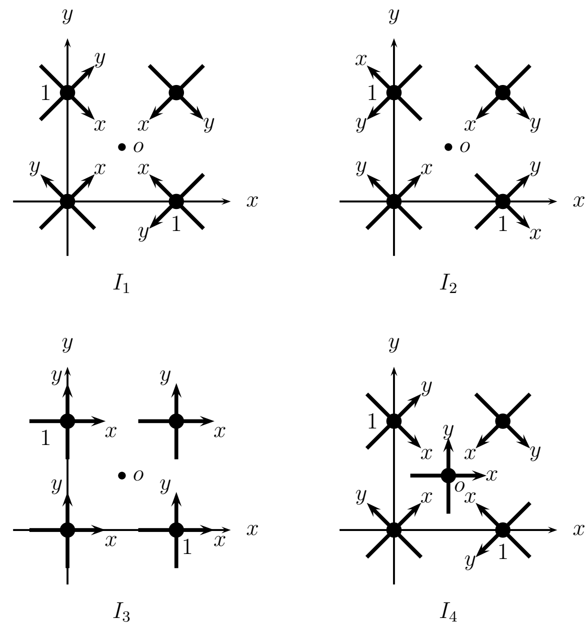

Figure 2: Illustration for Example 1.

First, consider $I_1 = \{(p_i, Z_i^1) | i = 1,2,3,4\}$, where $Z_i^1 = (o_i, d_i^1)$ and $d_i^1 = (1/2, 1/2)$ ($i = 1,2,3,4$). Rotation $\gamma^{(k)}$ of angle $k\pi/2$ around o maps I_1 to I_1 for $k = 0,1,2,3$. Thus $\sigma_o(I_1) = 4$.

Next, consider $I_2 = \{(p_i, Z_i^2) | i = 1,2,3,4\}$, where $Z_i^2 = (o_i, d_i^2)$, $d_1^2 = (1/2, 1/2)$, $d_2^2 = (3/2, -1/2)$, $d_3^2 = (1/2, 1/2)$ and $d_4^2 = (-1/2, 3/2)$. Rotation $\gamma^{(k)}$ of angle $k\pi$ around o maps I_2 to I_2 for $k = 0,1$. Thus $\sigma_o(I_2) = 2$.

Consider $I_3 = \{(p_i, Z_i^3) | i = 1,2,3,4\}$, where $Z_i^3 = (o_i, d_i^3)$, $d_1^3 = (0, 1/2)$, $d_2^3 = (1, 1/2)$, $d_3^3 = (1, 3/2)$ and $d_4^3 = (0, 3/2)$. Observe that no rotation γ of an angle in $(0, 2\pi)$ around o maps I_3 to I_3. Thus $\sigma_o(I_3) = 1$.

Finally, consider I_4, which is identical to I_1 except that there is a fifth robot at o. The reader can verify that regardless of the orientation of the local coordinate system of the robot at o, no rotation γ of an angle in $(0, 2\pi)$ around o maps I_4 to I_4. Thus $\sigma_o(I_4) = 1$. □

Let $\sigma(I) = \max_{o \in \mathbf{R}^2} \sigma_o(I)$ and call it the *symmetricity* of I. Symmetricity $\sigma(I)$ is well-defined, since I has a trivial rotation $\gamma_{o,0}$ of 0 radian about any point o that maps I to itself. If $\sigma(I) \geq 2$, then the center o that achieves $\sigma(I) = \sigma_o(I)$ is unique and is the center of $C(P(I))$, where for any point set S, $C(S)$ denotes the smallest enclosing circle of S. If $\sigma(I) = 1$, then though any point o can be used as the center of rotation, we adopt the convention to choose as o the center of $C(P(I))$, unless otherwise stated.

For $n \geq 2$ and a divisor m of n, let us define the following classes of patterns.

1. $\mathcal{F}_{\mathrm{FS,NO}}(n, m)$ = the set of patterns that n non-oblivious robots can form in the FSYNCH model from any initial configuration with symmetricity m.

2. $\mathcal{F}_{\mathrm{SS,NO}}(n, m)$ = the set of patterns that n non-oblivious robots can form in the SSYNCH model from any initial configuration with symmetricity m.

3. $\mathcal{F}_{\mathrm{FS,O}}(n, m)$ = the set of patterns that n oblivious robots can form in the FSYNCH model from any initial configuration with symmetricity m.

4. $\mathcal{F}_{\mathrm{SS,O}}(n, m)$ = the set of patterns that n oblivious robots can form in the SSYNCH model from any initial configuration with symmetricity m.

Our goal is to characterize these classes.

5 A necessary condition

We adopt the following convention: Any point is a regular 1-gon with center o for any o, a pair of points whose middle point is o is a regular 2-gon with center o. Furthermore, m points located at o may be viewed as forming a regular m-gon with center o.

The next proposition characterizes the structure of I in terms of $\sigma(I)$.

Proposition 1 [17] *Let $I = \{(p_i, Z_i) | 1 \le i \le n\}$ be a configuration. If $\sigma(I) = \sigma_o(I) = m$, then the n robots in $\mathcal{R}$ can be partitioned (not necessarily uniquely) into $k = n/m$ subsets of size m each, such that:*

1. *For any robots r_i and r_j in the same subset, there exists a rotation $\gamma_{o,\theta} \in \Gamma_o(I)$ such that $\gamma_{o,\theta}((p_i, Z_i)) = (p_j, Z_j)$.*

2. *The robots in the same subset form a regular m-gon with center o.*

3. *The robots in the same subset have identical sights in I.*

4. *Let Q be any of the subsets of m robots. For any point $q \in \mathbf{R}^2$, the set of points $\tilde{q}_i$ such that $[\tilde{q}_i]_{Z_i} = q$, over all robots $r_i \in Q$, forms a regular m-gon with center o.*

Suppose $\sigma(I(0)) = \sigma_o(I(0)) = m$ for initial configuration $I(0) = \{(p_i(0), Z_i) | 1 \le i \le n\}$. We observe the following, based on Proposition 1. At time 0 the robots are partitioned into $k = n/m$ subsets, each forming a regular m-gon with center o. In the FSYNCH model in which all robots become active simultaneously at all times, for any subset Q of m robots, the robots in Q observe the same sight and compute the same point q (using the same ψ). Each robot r_i in Q then moves to q in Z_i (which is $\tilde{q}_i$ in Z of Proposition 1), and their new positions at time 1 form a regular m-gon with center o. By repeating this argument, we observe that the robots in Q continue to form a regular m-gon with center o in all configurations reachable from $I(0)$. Therefore in the FSYNCH model, starting in $I(0)$, only those patterns F consisting of k regular m-gons with center o can be formed. The same conclusion holds for the SSYNCH model, in which all robots *can* become active simultaneously at all times. To summarize the above argument (see Lemma 1), let us introduce a measure ρ of the degree of symmetry in any point set.

Let $P = \{p_1, p_2, \ldots, p_n\}$ be a multiset of n points. For any point $o \in \mathbf{R}^2$, consider a partition of P into k regular m-gons with a common center o, where $k = n/m$. We call such a partition *regular*. (P has a trivial regular partition into n regular 1-gons with an arbitrary point as the common center.) Let $\rho_o(P)$ be the maximum m such that there is a regular partition of P into regular m-gons with center o, and define $\rho(P) = \max_{o \in \mathbf{R}^2} \rho_o(P)$. (In fact, if P has a regular partition into regular m-gons, where $m \ge 2$, then the common center coincides with the center of $C(P)$. When discussing a regular partition of P into n regular 1-gons, we adopt the convention to use the center of $C(P)$ as the common center.) Note that for any m, if P has a regular partition into regular m-gons, then m divides $\rho(P)$.

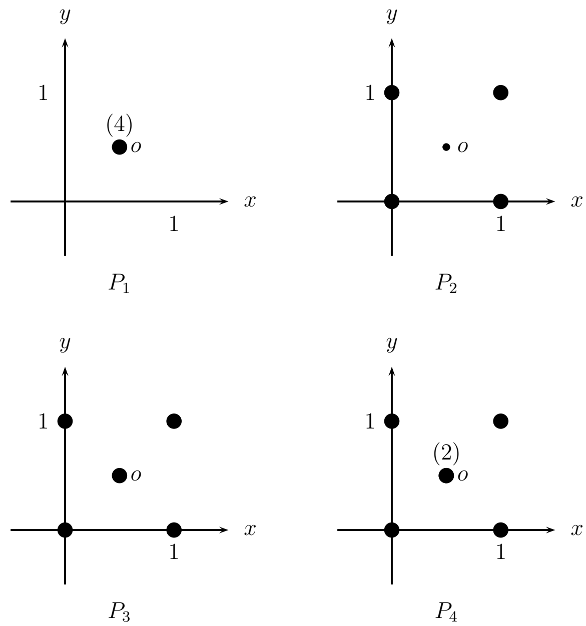

Figure 3: Illustration for Example 2. Numbers in parentheses indicate multiplicities.

Example 2 See Fig. 3 for illustration. First, consider P_1 consisting of four points whose coordinates are all $(1/2, 1/2)$. Any partition of P into $4/m$ m-sets for $m = 1, 2, 4$ is regular. Then $\rho(P) = \rho_o(P) = 4$, where $o = (1/2, 1/2)$.

Second, consider P_2 consisting of four points whose coordinates are $(0,0), (1,0), (1,1), (0,1)$. There are two regular partitions (besides the trivial one); partitioning into two regular 2-gons $\{\{(0,0),(1,1)\},\{(1,0),(0,1)\}\}$, and one regular 4-gon $\{\{(0,0),(1,0),(1,1),\ (0,1)\}\}$. Then $\rho(P) = \rho_o(P) = 4$, where $o = (1/2, 1/2)$.

Third, consider P_3 consisting of five points whose coordinates are $(0,0), (1,0), (1,1), (0,1), (1/2,1/2)$. Except for the trivial partition into regular 1-gons, there are no regular partitions, and hence $\rho(P) = \rho_o(P) = 1$, where $o = (1/2, 1/2)$ by the above convention.

Finally, consider P_4 consisting of six points whose coordinates are $(0,0), (1,0), (1,1), (0,1), (1/2,1/2), (1/2,1/2)$. There is a unique regular partition consisting of three 2-gons besides the trivial one; $\{\{(0,0),(1,1)\},\{(1,0),(0,1)\},\ \{(1/2,1/2),(1/2,1/2)\}\}$. Thus $\rho(P) = \rho_o(P) = 2$, where $o = (1/2, 1/2)$. □

Let us introduce, for any $n \geq 2$ and any divisor m of n:

$$\mathcal{P}(n, m) = \text{the set of patterns } F \text{ of } n \text{ points such that } m \text{ divides } \rho(F).$$

Then we have:

Lemma 1 [17] *For $n \geq 2$ and a divisor m of n,*

$$\mathcal{F}_{\mathrm{FS,NO}}(n, m), \mathcal{F}_{\mathrm{SS,NO}}(n, m), \mathcal{F}_{\mathrm{FS,O}}(n, m), \mathcal{F}_{\mathrm{SS,O}}(n, m) \subseteq \mathcal{P}(n, m).$$

That is, in both SSYNCH and FSYNCH models, there exists a function ψ that solves the formation problem for F starting from $I(0)$ only if $\sigma(I(0))$ divides $\rho(F)$.

Proof. See the above argument. □

We conclude this section by introducing the following notation. For any configuration I with robot positions $P(I)$, let $\rho_o(I) = \rho_o(P(I))$ and $\rho(I) = \rho(P(I))$, for representing the degree of symmetry in $P(I)$ ignoring Z_i's. Clearly, $\sigma_o(I)$ divides $\rho_o(I)$, and $\sigma(I)$ divides $\rho(I)$.

6 Pattern formation by non-oblivious robots

Lemma 1 applies to both oblivious and non-oblivious robots. The next lemma shows that non-oblivious robots can form all patterns mentioned in that lemma.

Lemma 2 [17] *For $n \geq 2$ and a divisor m of n,*

$$\mathcal{P}(n,m) \subseteq \mathcal{F}_{\mathrm{FS,NO}}(n,m), \mathcal{F}_{\mathrm{SS,NO}}(n,m).$$

That is, in both SSYNCH and FSYNCH models, there exists a non-oblivious function ψ that solves the formation problem for F starting from $I(0)$ if $\sigma(I(0))$ divides $\rho(F)$. (ψ does not depend on $I(0)$.)

Proof. We give only a proof outline. It suffices to consider the SSYNCH model, since any pattern that can be formed in the SSYNCH model starting from $I(0)$ can be formed in the FSYNCH model starting from $I(0)$ using the same function, i.e., $\mathcal{F}_{\mathrm{SS,NO}}(n,m) \subseteq \mathcal{F}_{\mathrm{FS,NO}}(n,m)$.

Since describing ψ purely as a function is not only tedious but also unintuitive, we instead describe the robots' moves under ψ. The reader is reminded that the robots can memorize what they observe since they are non-oblivious.

For simplicity of explanation, we assume that each robot r_i is located at the origin o_i of its coordinate system Z_i at time 0. The same result holds even without this assumption. Let us first introduce a basic scheme.

Broadcasting a directed line: Suppose that each robot r_i has (privately) chosen a directed line ℓ_i that passes through its initial position. The robots can simultaneously "broadcast" the locations and directions of $\ell_1, \ell_2, \ldots, \ell_n$, as follows. All robots r_i moves repeatedly along their respective lines ℓ_i in the positive direction. It can then be shown that if r_i has observed r_j, $j \neq i$, at four or more distinct locations, then r_j has observed r_i at two or more distinct locations, and hence, both r_i and r_j know each other's chosen directed lines. There are two issues here.

1. Since the robots are indistinguishable by their appearances, if two robots get "too close" to each other, then other robots (based on two sights they obtain at different times) may not be able to figure out their moves correctly. We cope with this by letting each r_i memorize the distance $a_i > 0$ (in Z_i) to its nearest neighbor when it becomes active for the first time and move at most distance $a_i/2^{k+1}$ in the k-th move. Then each r_i will stay in the interior of the $(a_i/2)$-neighborhood of its initial position, and thus all robots can correctly determine which robot has moved to which position even after they remain inactive for a long time.

2. If we allow r_i to simply stop moving as soon as it has observed r_j at four or more distinct locations, then r_j may not be able to observe r_i at four or more distinct locations. This means that r_j may never finish the broadcast. To cope with this, we let any robot r_i that has observed all other robots at four or more distinct locations change directions and move back to its initial position along ℓ_i. Robot r_j, $j \neq i$, when it observes that r_i has changed directions, knows that r_i has already seen r_j at four or more distinct locations and knows ℓ_j. To summarize, for $1 \leq i \leq n$, r_i repeatedly moves along ℓ_i in the positive direction, and changes directions and returns to its initial position along ℓ_i (disregarding the distance constraint in each step mentioned above) when for each $j \neq i$, either

   (a) r_i has seen r_j at four or more distinct locations, or

   (b) r_i observes that r_j has changed directions (r_i knows ℓ_j by the time this occurs).

□

Using the above scheme three times, the robots first broadcast their local coordinate systems.

Broadcasting Z_i: To broadcast $Z_i = (o_i, d_i)$, robot r_i broadcasts three directed lines one by one, first the x-axis, then the y-axis, and finally line L_i through o_i in direction $f(d_i)$, where d_i is the minimum distance between any two initial positions of the robots measured in Z_i, and for $x > 0$, $f(x) = (1 - 1/2^x) \times \pi/2$ is a monotonically increasing function with range $(0, \pi/2)$. Note that the three directed lines have distinct orientations. Specifically, robot r_i broadcasts its x-axis and returns to o_i as described above, and then starts broadcasting its y-axis. Eventually all robots finish broadcasting their x-axes and start broadcasting their y-axes. When the broadcast of the y-axes are finished, each r_i knows the initial positions of all robots (as the intersections of their x- and y-axes) and hence, knows the direction $f(d_i)$ of line L_i that it now broadcasts. Once again, each robot returns to its initial position when the broadcast is completed. □

At this moment, the robots know their initial configuration $I(0) = \{(p_i(0), Z_i) | 1 \leq i \leq n\}$. Thus every robot r_i can compute the sight $[P(0)]_{Z_j}$ of r_j, $j \neq i$, in $I(0)$ and $m = \sigma(I(0)) = \sigma_o(I(0))$, where o is the center of $C(P(0))$. Since the robot positions are all distinct in $I(0)$, using any fix total order over sights we can uniquely partition the set $\mathcal{R}$ of robots into $k = n/m$ *ordered* subsets $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_k$, each forming a regular m-gon with center o. (All robots can compute this ordered decomposition and memorize the index s of $\mathcal{R}_s$ to which they belong.) At the same time, since by assumption $m = \sigma(I(0))$ divides $\rho(F)$, and since the points in F are given as coordinates in Z, we can uniquely partition F into k *ordered* subsets $F_1, F_2, \ldots, F_k$, each forming a regular m-gon with a common center. (Although F may contain multiplicities, the decomposition is unique, in the sense that for each index s, the coordinates of the points in F_s are uniquely determined. Again, all robots can compute this ordered decomposition.) See Fig. 4 for illustration.

Based on these two ordered decompositions, for each s, $1 \leq s \leq k$, the robots in $\mathcal{R}_s$ move to suitable positions to form regular m-gon F_s. Obviously, there exists a scheme that computes such positions for any $I(0)$ and F such that $\sigma(I(0))$ divides $\rho(F)$. For instance, first we translate F so that the center of the regular decomposition $F_1, F_2, \ldots, F_k$ coincides with o. If $F_1, F_2, \ldots, F_n$ are all multiplicities of size m located at o, then all robots simply move to o. Otherwise, let s' be the smallest index such that $F_{s'}$ is not point o with multiplicity m. The robots in $\mathcal{R}_{s'}$ need not move any further, since they already

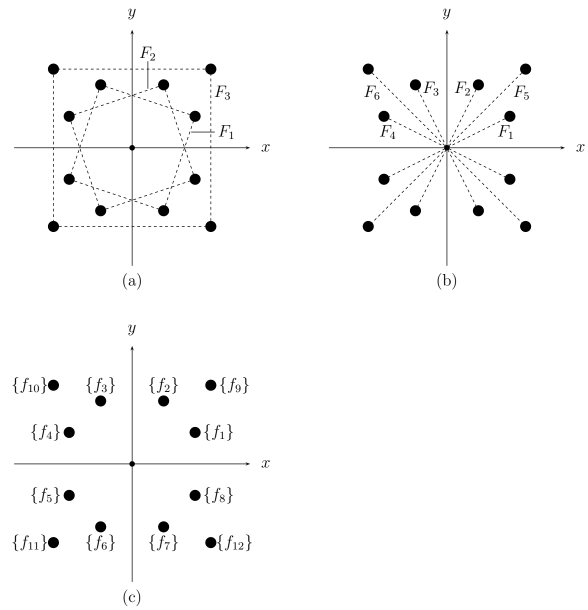

Figure 4: Sample ordered partitions of $F = \{f_1, f_2, \ldots, f_{12}\}$ with $\rho(F) = 4$ into subsets of size m; (a) $m = 4$, F_1, F_2, F_3, (b) $m = 2$, $F_1, F_2, \ldots, F_6$, (c) $m = 1$, $\{f_1\}, \{f_2\}, \ldots, \{f_{12}\}$.

form an m-gon similar to $F_{s'}$. Each robot in $\mathcal{R}_{s''}$, $s'' \neq s'$, computes the positions of the corners of $F_{s''}$ relative to the location of $F_{s'}$, and moves to the closest corner (breaking ties in some deterministic manner). □

By Lemmas 1 and 2, we obtain the following theorem that characterizes the class of geometric patterns that non-oblivious robots can form in the SSYNCH and FSYNCH models.

Theorem 1 [17] *For $n \geq 2$ and a divisor m of n,*

$$\mathcal{F}_{\mathrm{FS,NO}}(n, m) = \mathcal{F}_{\mathrm{SS,NO}}(n, m) = \mathcal{P}(n, m).$$

7 Pattern formation by oblivious robots

In the proof of Lemma 2, we took full advantage of the robots' non-obliviousness, namely, their ability to memorize sights they observed in the past. We now investigate what patterns can be formed by n oblivious robots that do not have such memory. It is helpful to first discuss the *point formation problem*, in which the given pattern F represents a single point of multiplicity n. This problem is often referred to in the literature as the *rendezvous problem* [8] [10].

First, let us consider the point *convergence* problem for $n = 2$, i.e., two oblivious robots r_1 and r_2 must converge to a single point (but they are not required to occupy the same point in finite time). Clearly the following oblivious function ψ achieves this goal in the SSYNCH model (and hence, in the FSYNCH model as well). (Again, we describe ψ as robots' actions.) Here, "a robot moves toward point p" means "a robot moves to the point p' closest to p that is reachable in one step." Of course, $p = p'$ if p is reachable in one step.

Function ψ_1: Each time r_i becomes active, it moves toward the midpoint of its current position and that of the other robot r_j. □

Note that function ψ_1 solves point formation for $n = 2$ in the FSYNCH model, since both robots always move toward the midpoint of their positions simultaneously. In contrast, ψ_1 does not solve point formation in the SSYNCH model, because if only one robot becomes active at every time instant, then the two robots executing ψ_1 will never occupy the same point. In fact, we have the following theorem (the claim for the case $m = 2$ was first proved in [17]).

Theorem 2 [17][20] *For $m = 1, 2$ and any pattern F describing a single multiplicity of size two, $F \notin \mathcal{F}_{\mathrm{SS,O}}(2, m)$. That is, in the SSYNCH model, there is no oblivious function ψ that solves point formation for $n = 2$.*

Proof. Suppose that there exists an oblivious function ψ that solves point formation for two robots r_1 and r_2. Note that since ψ is oblivious, the robots' moves depend only on their current sights.

We first show that there must exist a configuration $I = \{(p_i, Z_i) | i = 1, 2\}$ of distinct positions p_1 and p_2 such that either (1) ψ moves r_1 from p_1 to p_2, and r_2 from p_2 to p_2, or (2) ψ moves r_1 from p_1 to p_1, and r_2 from p_2 to p_1. (That is, ψ moves exactly one robot to the position of the other, if both robots become active simultaneously.) To see this, assume that such a configuration does not exist. Consider an activation schedule $\mathcal{A}$ under which r_1 and r_2, located at distinct positions p_1 and p_2 at time $t - 1$, respectively, occupy the same position q at time t. Now we show that we can modify $\mathcal{A}$ and obtain another activation schedule in which the robots never occupy the same position simultaneously. There are two cases.

Case 1: Both r_1 and r_2 are active at time $t - 1$ in $\mathcal{A}$.
By assumption, $q \neq p_1$ and $q \neq p_2$. So if only one robot, say r_1, becomes active at $t - 1$, then at time t, r_1 is located at q and r_2 at p_2, where $q \neq p_2$.

Case 2: Exactly one robot is active at $t - 1$ in $\mathcal{A}$.
Suppose that r_1 is active at $t - 1$ but r_2 is not. Then $q = p_2$. So if both robots become active at $t - 1$, then at time t, r_1 is located at p_2 and r_2 at some point q', where by assumption, $q' \neq p_2$.

Using this argument repeatedly, we can construct an infinite sequence of moves in which the robots never occupy the same position simultaneously. (We can do so in such a way that both robots become active infinitely many times, since either of the robots can be chosen to be inactive in Case 1.) So ψ does not solve point formation. This is a contradiction.

So, consider an initial configuration $I(0) = I$ in which r_1 and r_2 are located at p_1 and p_2, respectively, and ψ moves r_1 from p_1 to p_2, and r_2 from p_2 to p_2. See Fig. 5(a). (The argument for the case in which ψ moves r_2 to the positions of r_1 is similar.) Now, by modifying Z_1 suitably, we can construct another configuration I' having the same distribution $\{p_1, p_2\}$ as I in which r_1 and r_2 have the same sight, i.e., $[\{p_1, p_2\}]_{Z_1} = [\{p_1, p_2\}]_{Z_2}$, as shown in Fig. 5(b). Then in I', ψ moves r_1 from p_1 to p_1, and r_2 from p_2 to p_2. This means neither robot moves from its current position. Therefore ψ does not solve point formation. This is a contradiction. This completes the proof of the claim for $m = 2$,

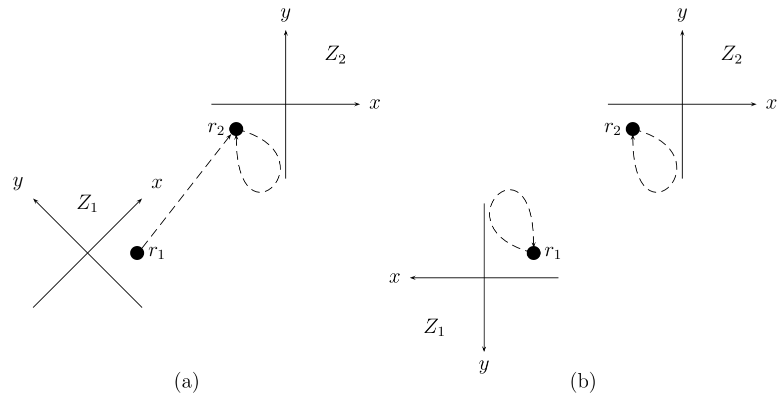

Figure 5: (a) In I, r_1 moves to r_2's position and r_2 does not move. (b) In I', neither r_1 nor r_2 moves.

since $\sigma(I') = 2$. We omit the proof for the case $m = 1$, which is slightly more technical. The reader is referred to [20]. □

Of course, by Theorem 1 two *non-oblivious* robots can form a point in the SSYNCH model, through a broadcast of their local coordinate systems. (Note that $\sigma(I(0)) = 1$ or 2 and $\rho(F) = 2$ for any initial distribution $I(0)$ of two robots and pattern F denoting a single point of multiplicity 2.) The following is a simpler solution for non-oblivious robots that does not require the broadcast [6]: Each robot, when it becomes active, memorizes its current position and moves toward the position of the other. Eventually, the robots get close enough so that they are mutually reachable in one step. Once this happens, if only one robot becomes active, then (according to the above action) that robot moves to the position of the other and forms a point. If both robots become active simultaneously, then they swap positions (recall that robots never collide with each other), but next time they become active (not necessarily simultaneously), they realize that in the previous step they were both active and saw each other's position. So they move to the midpoint of their previous positions and form a point.

Interestingly, as the next theorem shows, point formation for oblivious robots is solvable in the SSYNCH model if $n \geq 3$ (and hence in the FSYNCH model as well).

Theorem 3 *For any $n \geq 3$ and any divisor m of n, and for any pattern F describing a single multiplicity of size n, $F \in \mathcal{F}_{\mathrm{SS,O}}(n, m)$. That is, there exists an oblivious function ψ that solves point formation in the SSYNCH model for the case $n \geq 3$.*

Proof. Again, we describe ψ in terms of the robots' moves. The idea is the following. The robots, initially located at distinct positions, first create a single multiplicity (a point occupied by two or more robots) at some location p. Then the robots not located at p move toward p (and eventually reach there) without creating any other multiplicity. A point has been formed when all robots are at p.

A configuration with a single multiplicity can be obtained if the robots, each time they become active, move (or remain stationary) according to the following rules. (The rules collectively cover all situations that arise under various values of m.) Since the robots' actions are based only on their current positions, this strategy can be implemented by an oblivious function.

Case 1: $n = 3$. Let p_1, p_2 and p_3 be the positions of the robots. We write $|p_i p_j|$ to denote the distance between p_i and p_j. The line segment between p_i and p_j is denoted $\overline{p_i p_j}$.

> **1.1:** If p_1, p_2 and p_3 are collinear with p_2 in the middle, then the robots at p_1 and p_3 move toward p_2 while the robot at p_2 remain stationary. Then eventually a multiplicity is created at p_2.
>
> **1.2:** If p_1, p_2 and p_3 form an isosceles triangle with $|p_1 p_2| = |p_1 p_3| \neq |p_2 p_3|$, then the robot at p_1 moves toward the foot of the perpendicular drop from its current position to $\overline{p_2 p_3}$ in such a way that the robots do not form a regular triangle at any time, while the robots at p_2 and p_3 remain stationary. Then eventually the robots become collinear (Case **1.1**).
>
> **1.3:** If p_1, p_2 and p_3 form a triangle with distinct side lengths, say, $|p_1 p_2| > |p_1 p_3| > |p_2 p_3|$, then the robot at p_3 moves toward the foot of the perpendicular drop from its current position to $\overline{p_1 p_2}$, while the robots at p_1 and p_2 remain stationary. Then eventually the robots become collinear (Case **1.1**).
>
> **1.4:** If p_1, p_2 and p_3 form a regular triangle, then every robot moves toward the center of the triangle. If **1.4** continues to hold, then eventually either the robots meet at the center, or the triangle they form becomes non-regular (Case **1.2** or **1.3**).

Case 2: $n \geq 4$. Let C denotes the smallest enclosing circle of the current robot positions.

> **2.1:** If there is exactly one robot r in the interior of C, then r moves toward the position of the robot on the circumference of C that is closest to r (breaking ties in any deterministic manner). All other robots remain stationary. Then **2.1** continues to apply and r repeatedly moves toward that robot, eventually reaching there and creating a multiplicity.
>
> **2.2:** If there are two or more robots in the interior of C, then these robots move toward the center of C while all other robots remain stationary (so that the center of C remains unchanged). Then eventually at least two robots reach the center, creating a multiplicity.
>
> **2.3:** If there are no robots in the interior of C, then every robot moves toward the center of C. Then either (i) two or more robots occupy the center of C, creating a multiplicity, (ii) Case **2.1** or **2.2** applies, or (iii) **2.3** applies again for the smallest enclosing circle of the new robot positions that is now smaller than C.

□

The impossibility of point formation (or rendezvous) for two oblivious robots (Theorem 2) has motivated researchers to consider the problem further under additional assumptions on the robots' capabilities, such as a compass that may not be reliable [7] [14] [19]. For the benefit of the reader who is interested in this subject, a comprehensive survey of the recent progress on the rendezvous problem by Katayama and Yamashita [8] is included in the appendix.

At this point, the most important question that remains to be answered is the following: What is the class of patterns that n oblivious robots can form in the SSYNCH model (and hence in the FSYNCH model as well), starting from an initial configuration with symmetricity m? Surprisingly, it turns out that n oblivious robots can form any pattern that non-oblivious robots can, with a single exception of a point for the case $n = 2$ identified in Theorem 2.

Theorem 4 [20]

1. *For any $n \geq 3$ and any divisor m of n,*

$$\mathcal{F}_{\mathrm{FS,O}}(n, m) = \mathcal{F}_{\mathrm{SS,O}}(n, m) = \mathcal{P}(n, m).$$

2. *For $m = 1, 2$,*

$$\mathcal{F}_{\mathrm{FS,O}}(2, m) = \mathcal{P}(2, m)$$

and

$$\mathcal{F}_{\mathrm{SS,O}}(2, m) = \mathcal{P}(2, m) \setminus POINT_2,$$

where $POINT_2$ is the set of patterns describing a single multiplicity of size two.

That is, point formation for two robots in the SSYNCH model is the only problem that non-oblivious robots can solve but oblivious robots cannot. The proof of this recent result is quite involved, and we shall not attempt to present it in detail in this note. The interested reader is referred to [20]. The following is a brief summary of the argument.

Let $I(0)$ be an initial configuration of $n \geq 2$ oblivious robots located at distinct locations. Let F be a pattern of n points such that $m = \sigma(I(0))$ divides $\rho(F)$. If $n = 2$, then there are only two types of target patterns F, (i) a regular 2-gon (i.e., two points at distinct locations), and (ii) a point of multiplicity two (i.e., two points at the same location). The robots already form a regular 2-gon in $I(0)$, since they occupy distinct locations. The impossibility of forming a point in the SSYNCH model is given in Theorem 2.

Assume $n \geq 3$. Recall that $\rho(I(0))$ measures the degree of symmetry in $P(0)$ disregarding Z_i's, and $\sigma(I(0))$ divides $\rho(I(0))$. It turns out that if $\rho(I(0)) = 1$, then the robots can form any pattern F, as shown in the next lemma.

Lemma 3 *Let $F = \{f_1, f_2, \ldots, f_n\}$ be any pattern consisting of $n \geq 3$ points. There exists an oblivious function ψ for n robots to form F starting from any initial configuration $I(0)$ of distinct robot positions such that $\rho(I(0)) = 1$.*

Proof. The proof for the case $n = 3$ is relatively simple. Let $I(0) = \{(p_i, Z_i) | 1 \leq i \leq 3\}$, where p_1, p_2, p_3 are all distinct. In this case the condition $\rho(I(0)) = 1$ is equivalent to saying that p_1, p_2, p_3 do not form a regular triangle. In the following, for convenience of presentation, we continue to use p_1, p_2, p_3 to denote the positions of robots r_1, r_2, r_3, respectively, even after one or more moves. There should be no confusion.

Case F contains a multiplicity (i.e., $f_1 = f_2$, $f_2 = f_3$ or $f_1 = f_3$): Suppose one robot, say r_2, is equidistant from the other two, and hence (since $\triangle p_1p_2p_3$ is not a regular triangle) either $|p_1p_2| = |p_2p_3| < |p_1p_3|$ or $|p_1p_3| < |p_1p_2| = |p_2p_3|$ holds. Robot r_2 arbitrarily (but deterministically) selects a robot (r_1 or r_3), say r_1, and moves toward r_1 slightly, so that $|p_1p_2| < |p_2p_3| < |p_1p_3|$ or $|p_1p_3| < |p_1p_2| < |p_2p_3|$ holds after the move. Now we have reached a configuration in which no two edges of $\triangle p_1p_2p_3$ are of the same length. Assume without loss of generality that $|p_1p_2| < |p_2p_3| < |p_1p_3|$. Then robots r_1 and r_2 are uniquely identified as the closest pair, and r_1 is further identified as the one that forms the furthest pair with the third robot r_3. Now, robot r_2 moves toward r_1, while

r_1 and r_3 remain stationary. This move yields a new configuration in which r_1 and r_2 are closer while preserving the relation $|p_1p_2| < |p_2p_3| < |p_1p_3|$. Thus we can repeatedly (identify r_1 and r_2 as above and) execute this move until r_1 and r_2 share a position. When that happens, a formation with a multiplicity of size two has been reached. If F has a multiplicity of size two, then the current formation is similar to F. If gathering of all three robots is required, i.e., $f_1 = f_2 = f_3$, then the robot who recognizes that the other two have gathered at a single point moves to that point.

Case f_1, f_2 and f_3 are all distinct: Assume without loss of generality that $|f_1f_2| \leq |f_2f_3| \leq |f_1f_3|$. First, as in the previous case, the robots reach a configuration in which no two edges of $\triangle p_1p_2p_3$ are of the same length. Assume without loss of generality that $|p_1p_2| < |p_2p_3| < |p_1p_3|$. From now on robots r_1 and r_2 remain stationary. We associate r_1 (at p_1) and r_2 (at p_2) with f_1 and f_2, respectively, and move r_3 (in multiple steps if necessary) to a position p such that $\triangle p_1p_2p$ is similar to F. There are at most two candidates for p, and if there are two, then r_3 chooses the one closer (breaking the tie in some deterministic manner). While moving toward p, if its next location is not p, then r_3 always chooses its next location p_3 so that $|p_1p_2| < |p_2p_3| < |p_1p_3|$ continues to hold, so that r_3 will continue to be identified as the only robot that is allowed to move. By repeating this, r_3 eventually reaches p.

The following is an outline of the proof for the case $n \geq 4$. Using the assumption $\rho(I(0)) = 1$, we define a unique total order $r_1, r_2, \ldots, r_n$ on $\mathcal{R}$, based on their distances from the center of $C(P(0))$ *and* an ordering on their sights under artificially assigned "mutually visible" local coordinate systems. The points in F are also uniquely ordered as $f_1, f_2, \ldots, f_n$ based primarily on their distances from the center of $C(F)$ (with ties broken appropriately; see Fig. 4(c)). Then one by one and in the order $i = 1, 2, \ldots, n$, robot r_i moves to a point suitable to represent f_i. Since the robots are oblivious (and hence their action depends only on their current sights), this must be done in such a way that the ordering of the robots is maintained to a large extent and the partially formed portion of F is clearly recognizable. The entire process is quite complicated, and we omit the details here. □

Finally, let us give a brief outline of the case $\rho(I(0)) \geq 2$. If $\rho(I(0)) > \rho(F)$, then the robots first "reduce" the ρ value of their configuration by moving appropriately (a robot's move reflects its local coordinate system). Using the assumption that $\sigma(I(0))$ divides $\rho(F)$, we can show that eventually the robots reach a configuration J in which $m = \rho(J) = \rho_o(J)$ divides $\rho(F)$. Once such a configuration is reached, the robots partition themselves into $k = n/m$ ordered groups, $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_k$, each forming a regular m-gon with center o. Simultaneously, they partition the points in F into k ordered regular m-gons, $F_1, F_2, \ldots, F_k$ with a common center (see Fig. 4). Then, one by one and in the order $i = 1, 2, \ldots, k$, the robots in group $\mathcal{R}_i$ attempt to move to points suitable to represent F_i. Again, since the robots are oblivious, this must be done in such a way that the ordered partition of the robots and the partially formed portion of F is clearly recognizable. If during the operation the ρ value of the configuration changes, then new ordered partitions of $\mathcal{R}$ and F are computed and the robots continue to form the unfinished portion of F based on the new partitions (or Lemma 3 is used if ρ becomes 1). See [20] for details.

8 Concluding remarks

We gave a brief overview of the pattern formation problem for autonomous mobile robots in the plane and presented some of the key results known so far. There are several directions for future research. In the model adopted here, the robots have unlimited vision range, they do not block the vision of others, and their moves are instantaneous. Related work on the subject under different assumptions includes the case of robots with limited vision range [1], "fat" robots that block the vision of others [4], and the asynchronous (ASYNCH or CORDA) model in which the robots' moves are not instantaneous and a robot may be seen by others while it moves continuously to its target location [11]. The rendezvous problem has also been considered on a graph, where a robot hops from a vertex to another in discrete time steps [9].

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Appendix

A recent survey article on the point formation problem by Y. Katayama and M. Yamashita, "How to rendezvous," *Journal of the Society of Instrument and Control Engineers*, Vol. 46, No. 11, 2007, pp. 853–859, is reproduced in the following pages. We wish to thank the authors of the article and The Society of Instrument and Control Engineers for their kind permission.