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Bilateral practice of industrial mathematics in steel making process

Junichi Nakagawa and Masahiro Yamamoto

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Abstract. On the basis of our 10-year experiences of collaborating researches by mathematicians in academia and engineers from industry, we present a control problem in blast furnace as example of bilateral practice of industrial mathematics which is really used in factory. Moreover we discuss a desirable style of industrial mathematical collaboration by mathematicians and engineers.

Keywords. bilateral practice, industrial mathematics, steel making process, blast furnace,inverse heat conduction problem, method for real use

1. Introduction

The authors have been working with several joint research projects related with steel making processes since 2000. The first named author is an engineer and the second named author is a faculty member at department of mathematical sciences, and on respective different backgrounds and cultures, taking advantage of both sides, we have created and developed methods for problems in the steel making process which are for real use.

As joint researches involving both university and industry, needless to say, there is a traditional collaboration style by engineering faculties of universities and industry. On the other hand, in the USA and European countries such as England, France, Germany and Austria, the practice of industrial mathematics as bilateral collaboration by mathematicians and industry is recently very common and often supported substantially by governments, and through the "practice of industrial mathematics", the industrial partners have been able to develop more effective and comprehensive methods in solving their real industrial problems yielding economical merits, while mathematicians have been stimulated and motivated by realistic problems to gain new prospects and create new fields even in mathematics. As for a recent report on the activities related with industrial mathematics, see OECD report
http://www.oecd.org/dataoecd/31/19/42617645.pdf

However it seems that in Japan there are few such bilateral collaboration activites by mathematicians and industry.

Our trial for 10 years can be one example of the bilateral practice of the industrial mathematics, and we here discuss aspects of the industrial mathematics on the basis of our experiences.

As industrial mathematics, there are very various types of problems, but most of our problems have been concerned with what are called "inverse problems", where one is requested to evaluate the cause or detect directly invisible objects by available data. In this article, related to inverse problems, we discuss

(1) Section 2: How can the mathematics of the inverse problem be useful for the industry?

(2) Section 3: Case study; risk-management and innovation for operation of blast furnace

(3) Section 4: Our style of the biltareal practice of industrial mathematics

2. How can the mathematics of the inverse problem be useful for the industry?

"Inverse problem" contains tremendously various problems, and we can understand it as problem where we are required to determine objects which can not be recognized direcly, by means of our available measurement data. Therefore, as inverse problem, one can keep in mind various kinds of problems such as the medical diagnosis, the physical prospecting, and many problems in industry can be considered as inverse problems. Since the inverse problem is a determination problem of some states by incomplete data, the inverse problem commonly has the intrinsic instability. That is, even if noises in measurement data are small, huge deviations may occur in resulting solutions reconstructed by data.

In Section 3, related with industry, we discuss one state estimation problem in a blast furnace: determination of heat-flux in a deep interior of a blast furnace by temperature data taken near the surface, where the direct observation of the interior heat-flux is impossible by the size and physical conditions of the blast furnace, and we can use only measurements at far points. Moreover it is often that the industrial inverse problem is serious for the security and risk management. Because in the case of the blast furnace, it is crucial that by evaluated values of the heat-flux by a method of inverse problems, engineers should control the process in the furnace for gaining economical reasonability and keeping the security (e.g., avoiding melt-down of the hearth).

Here focusing on the inverse problem, we state five advantages of the industrial mathematics.

1. In view of mathematics, one can comprehensively review and reconsider specific solution methods which have been used individually for concrete problems in the real world. Mathematics offers general theories and so it is possible that under unified principles, we can uniformly treat several problems which look like very different and were inclined to be considered separatedly in different sections of industry. If one can introduce mathematical thinking manners, then it is expected to treat several problems simultaneously and share gained knowledge and techniques over one working section, which may yield saving for costs and time.

Because of the instability for the inverse problem, we need special techniques for overcoming the proper instability to obtain reasonably accurate solutions within noise level. Such instability and relevant methods for the inverse problem are studied already as mathematical subjects. Therefore, given a concrete inverse problem in indusrty, it is more reasonable to first clarify the problem in view of the general mathematical theory and then take into consideration related factors attached to the given problem, rather than to consider the given problem ad hoc according to different situations in industry.

As other example where a mathematical unified thinking manner is useful, we can refer to the inverse problem by Dirichlet-to-Neumann map which has been studied mathematically profoundly by the theory of partial differential equations. On the other hand, imporant problems in the medial diagnosis and the physical prospecting are described by the Dirichlet-to-Neumann map, and it is desirable for the engineer to keep the contact with mathematicians who can explain relevant mathematical theories and backgrounds in common languages. In reality, it seems that the researchers in the medical field and the physical prospecting have been working separetedly, but needless to say it is more effective that both reserachers can share the theoretical backgrounds, and for it mathematics can be a platform.

2. The thinking manners of mathematicians are less dependent on the existing hardwares and methodologies, compared with engineers, and can be expected to propose initiating ideas for flexible, non-conventional and innovating methods.

We note that for the realization of such initiating ideas for the real applications, the relevant collaboration by mathematician and engineers in industry is indispenable (see Section 4).

3. As is stated as the second advantage, the activity range of mathematicians is totally international. It is often that mathematicians keep international networks of human resources. If one industrial problem is given suitably (see Section 4), then a mathematician can invite members from ones human resources to activate their knowledge and solve the mission.

We need mathematicians with abilities for such managements, who can be a "right person" from the side of mathematics for the bilateral practice of industrial mathematics (see Section 3.3).

4. For industrial inverse problems, one can expect nice cost performances by the participation of mathematicians.

In the inverse problem for a blast furnace, the demanded mission is to create methods for data interpretation yielding reasonable estimation of interior heat-flux, where data have been already collected. For that inverse problem, we need not prepare new equipments for the measurement and rely on already collected data.

5. The mathematical specification may be useful for succession of the technology. There are many masters' arts which were obtained on the basis of traditions and experiences, and without such arts one can not manufacture products of high quality. In order to pass such traditional arts over generations, one has to try to clarify as quantitatively as possible. Therefore it is significant to learn such arts not only by experiences (although empirical learning is important), but also by clarifying the governing laws behind phenomena. For it, mathematics is a reliable tool.

3. Case study; risk-management and innovation for operation of blast furnace

A blast furnace controls a basic process for the steel making process, which produces molten iron from sintered ore and coke through deoxidizing process, and the inside temperature distribution is not uniform and very high. We are concerned with the basement of the furnace which is covered by special bricks, and the brick directly touches the process space whose temperature is about 1,500 degrees Celsius. The brickworks are robust against the high temperature and designed to protect the plant and avoid damages of the furnace. It is seriously necessary for us to estimate the heat-flux on the inner surface of the brickworks because the heat-flux can be a good index for secure controlled process. If the heat-flux irregularily behaves, then one has to reduce the activity level of the furnace. On the other hand, the reduction of the activity level implies the economical loss, and should be avoided as long as the safety is secured.

However the direct observation of the interior heat-flux is impossible by the size and structure of the furnace and the high temperature, so that a possible way is only to estimate them by temperature data near the outer surface of the furnace which are observed by thermocouples (Figure 4).

Occasionally irregular behaviour of such temperature data is observed and, considering such an abnormal state as omen for coming worse irregularity, the furnace engineers conventionally made a shutdown operation for reducing the furnace activity. Such unscheduled shutdown suspends the production of molten iron and it must continue over a few months for the safety.

Therefore important missions are:

(1) Can we reasonably estimate how the interior heat-flux behaves by available outer temperature data?

(2) Is there an effective mean for predicting such an abnormal period?

The first issue is concerned with an inverse problem called *Inverse Heat Conduction Problem*, and the second is related with the risk management in view of ecomonical reasonability.

Thus our task is the evaluation of heat-flux on molten iron surface by far-away data

Our joint research project is composed of 3 phases:

(1) construction of a mathematical model

(2) invention of a numerical method for the real use.

(3) fulfilment of industrial requests

3.1. Phase I: construction of a mathematical model

Inside the blast furnace, complex processes go with multi-phases involving solid and liquid phases under high temperature, and the modelling itself is a serious problem and may requests a long-term research. However, our main purpose is to find an index for a secure operation of the furnace by solving the corresponding inverse problem.

Thus for choosing a model, our strategy is

the choice of a minimum necessary model equation.

In order to make sure whether our chosen model is a minumim necessary model equation, we should go to the next phases and repeatedly compare numerical results with real data. In other words, we should not presume that our firstly chosen model is the minimum by thinking only the first phase, but by means of the total reviews through the whole phases. For example, our chosen minimum model must fulfill also requests from real works such as short CPU time, programmes not requesting special computer facilities (e.g., without super-computer). Otherwise the numerical method on the basis of the model equation can not be used on site by engineers.

In the present case, our minimum necesssary model equation is a simple one-dimensional heat equation:

$$u_t(x,t) = \alpha u_{xx}(x,t), \quad 0 < x < \ell,\ t > 0,$$

where a constant $\alpha > 0$ is the thermal diffusion coefficient.

At this point, analysts may be able to criticize that our model is too simple, because the blast furnace is not one-dimensional, the high temperature needs some nonlinear dependency of α on u, etc. However again we should note that the suitability of the choice of the model can be well judged by the final outputs through the whole phases.

Thus our mathematical formulation of the inverse heat conduction problem is:

$$u_t(x,t) = \alpha u_{xx}(x,t), \quad 0 < x < \ell,\ t > 0. \tag{3.1}$$

$$u(\ell,t) = h(t), \qquad t > 0. \tag{3.2}$$

$$u_x(\ell,t) = g(t), \qquad t > 0. \tag{3.3}$$

Here we consider only the depth direction from the outer boundary of the furnace ($x = \ell$) to the molten steel surface ($x = 0$), and take the one-dimensional heat conduction in the brick. The functions $g(t)$ and $h(t)$ are data observed by the thermocouples. Then we can state our inverse problem:

Inverse Heat Conduction Problem. Determine

$$f(t) = -u_x(0,t), \qquad 0 < t < T$$

by $g(t), h(t)$, $0 < t < T$.

Here we note that initial data $u(x,0)$, $0 < x < \ell$ is also unknown.

Mathematical discussions on the inverse heat conduction problem: In order to understand the intrinsic instability which is a special character of the inverse problem, we consider a more simplified problem. That is, first we consider the following problem: Find $u(x,t)$, $x > 0$, $t > 0$ such that

$$\begin{cases} u_t(x,t) = u_{xx}(x,t), & x > 0,\ t > 0, \\ u(x,0) = a(x), & x > 0, \\ u_x(0,t) = -f(t), & t > 0 \end{cases} \tag{3.4}$$

for given $a(x)$ and $f(t)$.

This is a classical initial/boundary value problem and it is well-known that there exists a unique solution u to (3.4) within a suitable class (e.g., Cannon [3], Itô [9], Widder [15]), and the stability is proved: if data a and f change slightly, then the resulting solution u deviates accordingly small in suitable choices of norms measuring the changes and deviations.

On the other hand, our problem is of a different kind from (3.4) and we can state as follows:

Simplified inverse heat conduction problem: Let $a > 0$ be a fixed point. Determine $f(t) = -u_x(0,t)$, $t > 0$ by $u(a,t)$, $t > 0$ with

$$\begin{cases} u_t(x,t) = u_{xx}(x,t), & x > 0,\ t > 0, \\ u(x,0) = 0, & x > 0. \end{cases} \tag{3.5}$$

For this problem, we can prove the uniqueness: Assume that u and u_1 (in a suitable class) satisfies (3.5). Then $u(a,t) = u_1(a,t)$, $t > 0$ implies $\frac{\partial u}{\partial x}(0,t) = \frac{\partial u_1}{\partial x}(0,t)$, $t > 0$ (e.g., Cannon [3], Isakov [8]). However unlike the initial/boundary value problem (3.4), the existence and the stability in finding f do not hold. That is, there does not necessarily exist a solution f satisfying (3.5) and $u(a,t) = \widetilde{f}(t)$ for a given $\widetilde{f}$, and for u satisfying (3.5), the smallness of data $u(a,t)$ does not imply the smallness of $u_x(0,t)$, $t > 0$.

That is, the inverse heat conduction problem is ill-posed, although it is practically important. As for detailed information of the ill-posed problems, see for example [3], [8], [10], [11]. Here we intuitively explain why the stability does not hold. By [3], [9] for example, within smooth functions, we can have

$$u(x,t) = \int_0^t \frac{1}{\sqrt{\pi(t-s)}} e^{-\frac{x^2}{4(t-s)}} f(s)ds,\ x > 0,\ t > 0,$$

where $f(t) = -u_x(0,t)$. Therefore

$$u(a,t) = \int_0^t \frac{1}{\sqrt{\pi(t-s)}} e^{-\frac{a^2}{4(t-s)}} f(s)ds, \quad t > 0.$$

Therefore the inverse heat condution problem is to solve the above integral equation with respect to $f(t)$ from $u(a,t)$. Here the kernel function

$$\frac{1}{\sqrt{\pi(t-s)}} e^{-\frac{a^2}{4(t-s)}}$$

is very smooth in t, s and, any singularity of $f(t)$ does not appear in the data $u(a,t)$. In other words, in the inverse heat conduction problem, we have to reconstruct characteristic behaviour of heat flux $f(t)$ from smoothed data $u(a,t)$ and our solution is de-filtering. Thus we can not expect the stabiliy for the inverse heat conduction problem.

Furthermore in our case, we can not know also the initial value. Because we have to consider an on-line test for the blast furnace which usually should be operated for years without any breaks, and so it is impossible to assume the initial temperature distribution.

As one topic for mathematical analysis for the inverse heat condution problem, we refer to the conditional stability. That is, in spite of the instability, we can restore the stability within suitable a priori boundedness assumptions of unknown functions. The conditional stability is needed also for guaranteeing the quailty of numerical methods (e.g., Cheng and Yamamoto [4]). We will omit the details concerning analytical methods for obtaining the conditional stability and refer e.g., to a survey paper Yamamoto [16]. Such studies for the conditional stability need theories of partial differential equations and it is possible that mathematicians can make meaningful contribution to the practice of industrial mathematics.

3.2. Phase II: invention of a numerical method for the real use

For creating numerical methods for the real use, we have to keep in mind:

(1) Choice of minimum necessary model equation

(2) Fulfilling requests from real worksF short CPU time, not special programmes (e.g., without super-computers)

(3) Demanded accuracy VS realizable observation accuracy: Since the accuracy of observation data can not be improved drastically, the numerical method requesting high accurate data, is not helpful, and we need a numerical method producing accordingly reasonable results even with low accurate data.

(4) Theoretically consistent studies VS convenient method which workers at factories can use. We should not sacrifice the mathematical rigour.

In view of the above four points, we have created a numerical method for our inverse heat conduction problem and here we do not discuss the details. See Wang, Cheng, Nakagawa and Yamamoto [14]. There are already several numerical methods for various inverse problems for heat conduction and we can refer to Alifanov [1], Beck [2], Cannon [3], Hào [7], Murio [12] as monographs and see also Takeuchi [13]. The existing methods for the inverse heat conduction problem require initial data. Since in our case initial data are not available, we need special cares.

Our method relies on the eigenfunction expansions of the solution to the initial/boundary value problem for (3.1) with boundary values $-u_x(0,t) = f(t)$, $u_x(\ell,t) = g(t)$ and initial value $u_0(x)$:

$$u(x,t) = \sum_{n=0}^{\infty} A_n(x) e^{-\lambda_n t} + \int_0^t G(t-s,x,0) f(s)ds + \int_0^t G(t-s,x,\ell) g(s)ds, \quad 0 \le x \le \ell. \tag{3.6}$$

Here we set

$$\lambda_n = \alpha \frac{n^2\pi^2}{\ell^2}, \quad n \ge 0,$$

$$G(t,x,y) = \frac{2}{\ell} \sum_{n=1}^{\infty} e^{-\lambda_n t} \cos\frac{n\pi}{\ell}x \cos\frac{n\pi}{\ell}y + \frac{1}{\ell},$$

and

$$A_0(x) = \frac{1}{\ell}\int_0^\ell u_0(y)dy,$$

$$A_n(x) = \frac{2}{\ell}\left(\int_0^\ell u_0(y)\cos\frac{n\pi}{\ell}y dy\right)\cos\frac{n\pi}{\ell}x.$$

(e.g., [3], [9]).

For numerical solutions, we replace the infinite series in (3.6) by a finite sum, and it is crucial how many terms should be taken. Noting that the exponential factors of the terms with larger n are smaller, by means of the conditional stability analysis we can establish a guideline for numbers of terms which gives quasi-optimal numerical results for data with noises. Here for the qualification of the numerical method, the mathematics is helpful. Moreover by the conditional stability, we can evaluate errors caused by neglecting initial data. Since by $\lambda_n > 0$ the first term of (3.6) decays as t is large faster than the rest terms, the influence of the initial data can be negligible for large t and we need estimates for such t allowing us to neglect the contribution of the first term of (3.6).

Finally we have to stably solve the resulting linear system from the above truncated expansion of the solution, and the linear system is ill-conditioned, which easily causes the numerical instability, and we need a special stabilizing method which is called the regularization (e.g., [5], [6]).

We show one result of the numerical simulation. Figure 1 displays temperature data taken by two thermocouples which are set at different depths in a refractory material. We divide a temperature difference at the two depths by the distance between the two thermocouples and the ratio is used as surface heat-flux $u_x(\ell, t)$. Figure 2 indicates an artifically set heat-flux on the interior surface corresponding to $x = 0$ causing data in Figure 2. Figure 3 shows a numerical result by our method, compared with the existing methods. Beck's method is widely used and is a variational method (e.g., [2]) and near at switching times (see Figure 2) of heat-flux profile at $x = 0$, the reconstructed values deviate from the exact values larger than by our method. The pseudo-stationary method was used once, which simply uses the linear interpolation of the heat-flux with respect to x, and neglects the time evolution of the temperature distribution, so that the numerical performaces are of course bad. Our method is based on the Fourier series (see (3.6)) and so bothered with a Gibbs phenomenon (e.g., [17]): see the over-estimated or under-esimated values at three swithing times of inner heat-flux. However our method calculates values which are quite close to the set values.

3.3. Phase III: fulfilment of industrial requests

The final phase is crucial for transferring the created method to the real use because our mission can not be completed with good numerical simulations, but we have to check only whether our created method

(1) can treat real data,

(2) can provide manufacturing principles from the economical viewpoint.

This phase is not only under the theoretical thinking, but also by taking into consideration many elements from the reality at factory. The execution of this phase is highly case by case and difficult to explain generally. However the following points are indispensable.

(1) Right persons from both the mathematical commnunity and the industry.

(2) Continuous dialogues between mathematicians and industry through the right persons.

Here the right person from mathematics means a coordinator who is a mathematician, can work as translater between mathematical and industrial languages, and not only understands the requirements of industry/business (timeframe and desired ouputs in view of industry), but also holds strong leadership within research group/networks. On the other hand, the right person from industry means a coordinator who is a member of the partner company, and understands towards time frame/scheduling of the academia side. For example, it takes a long period to translate real world problems into the mathematical language, and, even for bright mathematicians, it may take substantial amount of time for solving issues practically. Moreover the coordinator from industry should understand the significance of fundamental researches of various mathematics, because one can not predict which mathematics will be useful.

Through the above three phases, as Figure 4 describes, our created evaluation method shows that before the anomaly period, huge oscillation of heat-flux on the molten iron surface appears and can not be recognized by directly available data. Thus such a huge oscillation pattern can act as effective index for predicting the anomaly and taking timely remedy control, which gains the economical effect. To sum up, our method which has been created by the bilateral practice of industrial mathematics, has turned to be a real guideline for a manufacturing process.

4. Our style of the bilateral practice of industrial mathematics

Figure 5 indicates our style of the collaboration with engineers and mathematicians in the case of Nippon Steel and the University of Tokyo. By the right persons from mathematics and industry, we have formed international task force teams made up of faculty members, post-doctoral fellows and doctor course students. Team members are selected flexibly according to the characteristics of the problems. In our collaboration, we consider six phases and at each phase, right persons (=coordinators) are indispensable. The first phase is "intuition and expertise" from industry. Intuition and expertise can be done exclusively by insight based on observations of phenomena in the manufacturing process. The second is "communication" between mathematicians and engineers. The communication involves bilateral translations: the translation of phenomena into the mathematical language and the adjustment of mathematics into phenomena. Engineers in industry need to view the real problem on site in physical words and offer possible minimum necessary model equations to mathematicians. Mathematicians explore the underlying mathematics which is applicable to the model equations. This forum for communication through the interpretation of phenomena is extremely important for the succeeding bilateral practice. The third is the construction of "logical path." This corresponds to the extraction of mathematical principles from phenomena and reasonable explanations of the phenomena. It is needless to say that better communication can create a more effective logical path. The fourth is "analysis of data." This means reasonable and quantitative interpretations of observations. The fifth is "manufacturing theory." This means the integration of logical paths from viewpoints of operation and economic rationality, and completes the transfer of the developed mathematical methods by the project into the real use. The bilateral practice of industrial mathematics discussed in Section 3 has reached this phase. The sixth is "activation to mathematics." Motivation for mathematicians has launched new mathematical research fields, and as the ideal form of the bilateral practice of industrial mathematics, we can expect such postive feedback also to the mathematics itself.

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Junichi Nakagawa
Mathematical Science & Technology Research Group, Advanced Technology Research Laboratories, Technical Development Bureau, Nippon Steel Corporation, 20-1 Shintomi Futtsu Chiba, 293-8511, Japan
E-mail: nakagawa.junichi(at)nsc.co.jp

Masahiro Yamamoto
Department of Mathematical Sciences, The University of Tokyo, Komaba Meguro Tokyo 153-8914 Japan
E-mail: myama(at)ms.u-tokyo.ac.jp

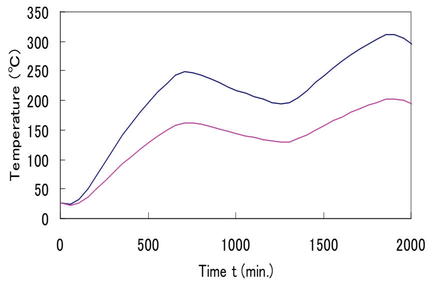

Figure 1: Time history of Temperature for numerical experiment

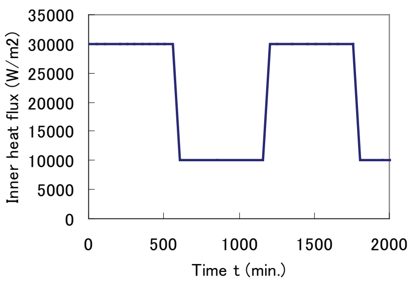

Figure 2: Set value of inner heat flux

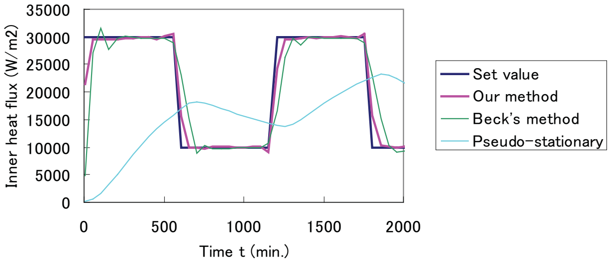

Figure 3: Numerical experiment results

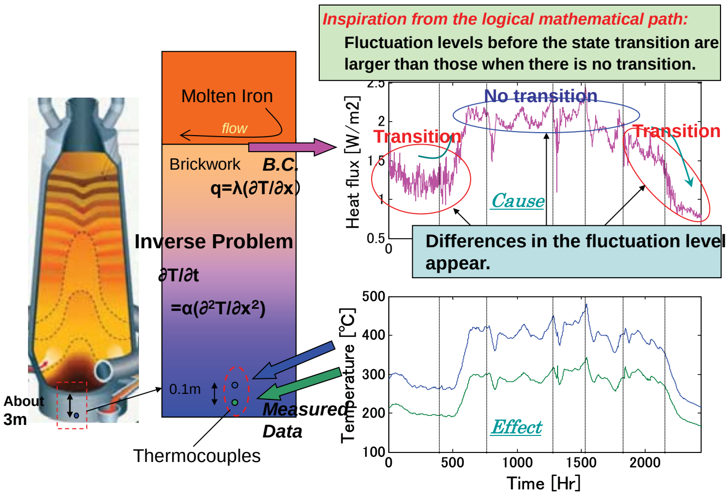

Figure 4: Reconstruction of Causal Quantity by IHCP

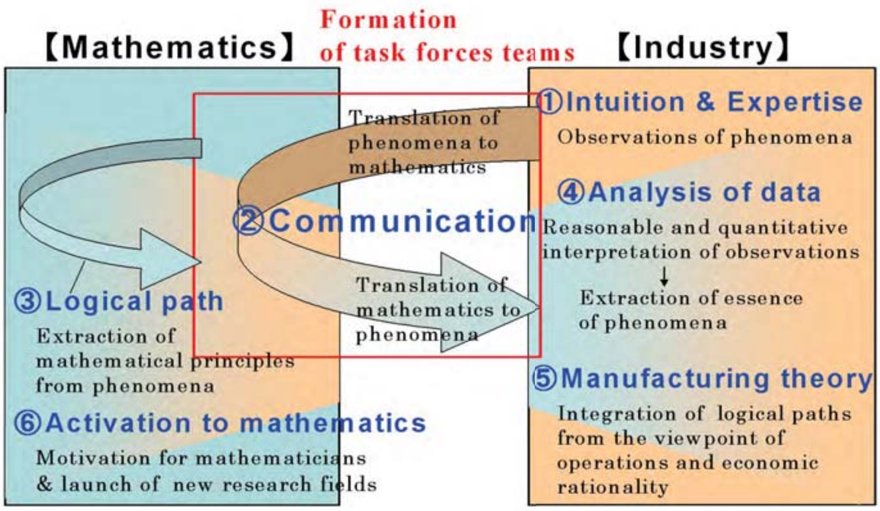

Figure 5: Collaboration with engineers and mathematicians in the case of Nippon Steel and the University of Tokyo