Recent progress in value
distribution of the hyperbolic
Gauss map

Y. Kawakami

MI 2009-24

(Received July 23, 2009)
RECENT PROGRESS IN VALUE DISTRIBUTION OF THE HYPERBOLIC GAUSS MAP

YU KAWAKAMI

ABSTRACT. We give a brief survey of our work on value distribution of the hyperbolic Gauss map. In particular, we define algebraic class for constant mean curvature one surfaces in the hyperbolic three-space and give a ramification estimate for the hyperbolic Gauss map of them. Moreover, we also give an effective estimate for the number of exceptional values of the hyperbolic Gauss maps of flat fronts in the hyperbolic three-space.

INTRODUCTION

One revealed the geometric meaning of the best possible upper bound of the number of exceptional values (for more precisely, defects) of meromorphic functions by using the Nevanlinna theory. In fact, Ahlfors [1] showed that the geometric meaning of the best possible upper bound “2” for the number of exceptional values of non-constant meromorphic functions on the complex plane $\mathbb{C}$ is the Euler number of the Riemann sphere. Moreover, Ahlfors [1] also showed that the upper bound of the number of exceptional values (defects) of non-constant holomorphic maps from $\mathbb{C}$ to a closed Riemann surface $\bar{M}$, corresponds to the Euler number of $\bar{M}$, that is, the target space of the maps.

On the other hand, the author, Kobayashi and Miyaoka [9] refined the Osserman argument [19, 20] and gave a ramification estimate for the Gauss map of pseudo-algebraic and algebraic minimal surfaces in the Euclidean three-space $\mathbb{R}^3$ recently. It also provided new proofs of the Fujimoto [5] and the Osserman theorems for these classes and revealed the geometric meaning behind it. We [10] also gave such an estimate for them in the Euclidean four-space $\mathbb{R}^4$.

The purpose of this survey is to give the geometric meaning of the upper bound of a ramification estimate for the hyperbolic Gauss map of non-flat algebraic constant mean curvature one (CMC-1, for short) surfaces and an effective estimate for the number of exceptional values of the hyperbolic Gauss maps for flat fronts in the hyperbolic three-space $\mathbb{H}^3$. In Section 1, we recall some fundamental facts and notations about CMC-1 surfaces in $\mathbb{H}^3$. In particular, using the notion of “duality”, we give the definition of algebraic CMC-1 surfaces. In Section 2, we give a ramification estimate for the hyperbolic Gauss map of algebraic CMC-1 surfaces and reveal geometric meaning behind it. In Section 3, after reviewing some notations and properties of flat fronts in $\mathbb{H}^3$, we give an
effective estimate for the number of exceptional values of the hyperbolic Gauss maps for this class.

The author thanks the organizers of The 16th Osaka City University International Academic Symposium 2008 “Riemann Surfaces, Harmonic Maps and Visualization”, in particular Professor Yoshihiro Ohnita, for the opportunity to talking and writing our study. The author also thanks Professors Ryoichi Kobayashi, Pablo Mira, Reiko Miyaoka, Wayne Rossman, Masaaki Umehara and Kotaro Yamada for their useful advice and comments.

1. Preliminaries

First, we recall some basic facts and notations on CMC-1 surfaces in $H^3$. Let $\mathbb{R}^4_1$ be the Lorentz-Minkowski 4-space with the Lorentz metric

$$\langle (x_0, x_1, x_2, x_3), (y_0, y_1, y_2, y_3) \rangle = -x_0 y_0 + x_1 y_1 + x_2 y_2 + x_3 y_3. \quad (1.1)$$

Then the hyperbolic 3-space is

$$H^3 = \{ (x_0, x_1, x_2, x_3) \in \mathbb{R}^4_1 \mid - (x_0)^2 + (x_1)^2 + (x_2)^2 + (x_3)^2 = -1, x_0 > 0 \}$$

with the induced metric from $\mathbb{R}^4_1$, which is a simply connected Riemann 3-manifold with constant sectional curvature $-1$. We identify $\mathbb{R}^4_1$ with the set of $2 \times 2$ Hermitian matrices $\text{Herm}(2) = \{ X \mid X^* = X \}$ by

$$(x_0, x_1, x_2, x_3) \leftrightarrow \begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}, \quad (1.2)$$

where $i = \sqrt{-1}$. In this identification, $H^3$ is represented as

$$H^3 = \{ a a^* \mid a \in SL(2, \mathbb{C}) \} \quad (1.3)$$

with the metric

$$\langle X, Y \rangle = -\frac{1}{2} \text{trace} (X \tilde{Y}), \quad \langle X, X \rangle = -\det(X),$$

where $\tilde{Y}$ is the cofactor matrix of $Y$. The complex Lie group $PSL(2, \mathbb{C}) := SL(2, \mathbb{C})/\{ \pm \text{id} \}$ acts isometrically on $H^3$ by

$$H^3 \ni X \longmapsto a X a^*, \quad (1.4)$$

where $a \in PSL(2, \mathbb{C})$.

There exists a representation formula for CMC-1 surfaces in $H^3$ as an analogy of the Enneper-Weierstrass representation formula for minimal surfaces in $\mathbb{R}^3$.

**Theorem 1.1** (Bryant [2], Umehara and Yamada [23]). Let $\tilde{M}$ be a simply connected Riemann surface with a reference point $z_0 \in \tilde{M}$. Let $g$ be a meromorphic function and $\omega$ be a holomorphic 1-form on $\tilde{M}$ such that

$$ds^2 = (1 + |g|^2)^2 |\omega|^2 \quad (1.5)$$

is a Riemannian metric on $\tilde{M}$. Take a holomorphic immersion $F = (F_{ij}) : \tilde{M} \to SL(2, \mathbb{C})$ satisfying $F(z_0) = \text{id}$ and

$$F^{-1} dF = \begin{pmatrix} g & -g^2 \\ 1 & -g \end{pmatrix} \omega. \quad (1.6)$$
Then \( f : \tilde{M} \to \mathcal{H}^3 \) defined by
\[
f = FF^*
\] (1.7)
is a CMC-1 surface and the induced metric of \( f \) is \( ds^2 \). Moreover, the second fundamental form \( h \) and the Hopf differential \( Q \) of \( f \) are given as follows:
\[
h = -Q - \overline{Q} + ds^2, \quad Q = \omega dg.
\] (1.8)

Conversely, for any CMC-1 surface \( f : \tilde{M} \to \mathcal{H}^3 \), there exist a meromorphic function \( g \) and a holomorphic 1-form \( \omega \) on \( \tilde{M} \) such that the induced metric of \( f \) is given by (1.5) and (1.7) holds, where the map \( F : \tilde{M} \to SL(2, \mathbb{C}) \) is a holomorphic null ("null" means \( \det (F^{-1}dF) = 0 \)) immersion satisfying (1.6).

**Remark 1.2.** Following the terminology of [23], \( g \) is called a secondary Gauss map of \( f \). The pair \((g, \omega)\) is called Weierstrass data of \( f \), and \( F \) is called a holomorphic null lift of \( f \).

Let \( f : M \to \mathcal{H}^3 \) be a CMC-1 surface of a (not necessarily simply connected) Riemann surface \( M \). Then the holomorphic null lift \( F \) is defined only on the universal cover \( \tilde{M} \) of \( M \). Thus, the Weierstrass data \((g, \omega)\) is not single-valued on \( M \). However, the Hopf differential \( Q \) of \( f \) is well-defined on \( M \). By (1.6), the secondary Gauss map \( g \) satisfies
\[
g = -\frac{dF_{12}}{dF_{11}} = -\frac{dF_{22}}{dF_{21}}, \quad \text{where } F(z) = \left( \begin{array}{cc} F_{11}(z) & F_{12}(z) \\ F_{21}(z) & F_{22}(z) \end{array} \right). \tag{1.9}
\]
The hyperbolic Gauss map \( G \) of \( f \) is defined by,
\[
G = \frac{dF_{11}}{dF_{21}} = \frac{dF_{12}}{dF_{22}}. \tag{1.10}
\]

By identifying the ideal boundary \( S^2_\infty \) of \( \mathcal{H}^3 \) with the Riemann sphere \( \mathbb{C} \cup \{\infty\} \), the geometric meaning of \( G \) is given as follows (cf. [2]): The hyperbolic Gauss map \( G \) sends each \( p \in M \) to the point \( G(p) \) at \( S^2_\infty \) reached by the oriented normal geodesics of \( \mathcal{H}^3 \) that starts at \( f(p) \). In particular, \( G \) is a meromorphic function on \( M \).

The inverse matrix \( F^{-1} \) is also a holomorphic null immersion, and produce a new CMC-1 surface \( f^\sharp = F^{-1}(F^{-1})^* : \tilde{M} \to \mathcal{H}^3 \), called the dual of \( f \) [26]. By definition, the Weierstrass data \((g^\sharp, \omega^\sharp)\) of \( f^\sharp \) satisfies
\[
(F^\sharp)^{-1}dF^\sharp = \begin{pmatrix} g^\sharp & -(g^\sharp)^2 \\ 1 & -g^\sharp \end{pmatrix} \omega^\sharp. \tag{1.11}
\]
Umehara and Yamada [26, Proposition 4] proved that the Weierstrass data, the Hopf differential \( Q^\sharp \), and the hyperbolic Gauss map \( G^\sharp \) of \( f^\sharp \) are given by
\[
g^\sharp = G, \quad \omega^\sharp = -\frac{Q}{dG}, \quad Q^\sharp = -Q, \quad G^\sharp = g. \tag{1.12}
\]
So this duality between \( f \) and \( f^\sharp \) interchanges the roles of the hyperbolic Gauss map and secondary Gauss map. We call the pair \((G, \omega^\sharp)\) the dual Weierstrass data of \( f \). Moreover, these invariants are related by
\[
S(g) - S(G) = 2Q, \tag{1.13}
\]
where \( S(\cdot) \) denotes the Schwarzian derivative
\[
S(h) = \left[ \left( \frac{h''}{h'} \right)' - \frac{1}{2} \left( \frac{h''}{h'} \right)^2 \right] dz^2, \quad \left( ' = \frac{d}{dz} \right)
\]
with respect to a complex local coordinate $z$ on $M$. By Theorem 1.1 and (1.12), the induced metric $ds^{22}$ of $f^2$ is given by

$$ds^{22} = (1 + |g^2|^2)|\omega|^2 = (1 + |G|^2)^2\left|\frac{Q}{dG}\right|^2.$$  

(1.14)

We call the metric $ds^{22}$ the dual metric of $f$. There exists the following linkage between the dual metric $ds^{22}$ and the metric $ds^2$.

**Lemma 1.3** (Umehara-Yamada [26], Z. Yu [27]). The Riemannian metric $ds^{22}$ is complete (resp. nondegenerate) if and only if $ds^2$ is complete (resp. nondegenerate).

Since $G$ and $Q$ are single-valued on $M$, the dual metric $ds^{22}$ is also single-valued on $M$. So we can define the dual total absolute curvature

$$TA(f^2) := \int_M (-K^2) dA^2 = \int_M \frac{4|dG|^2}{(1 + |G|^2)^2},$$

where $K^2(\leq 0)$ and $dA^2$ are the Gaussian curvature and the area element of $ds^{22}$ respectively. Note that $TA(f^2)$ is the area of $M$ with respect to the (singular) metric induced from the Fubini-Study metric on the complex projective line $\mathbb{P}^1(\mathbb{C}) (= \mathbb{C} \cup \{\infty\})$ by $G$. When the dual total absolute curvature of a complete CMC-1 surface is finite, the surface is called an algebraic CMC-1 surface.

**Theorem 1.4** (Bryant, Huber, Z. Yu). An algebraic CMC-1 surface $f: M \rightarrow \mathcal{H}^3$ satisfies:

(i) $M$ is biholomorphic to $\bar{M}_\gamma \setminus \{p_1, \ldots, p_k\}$, where $\bar{M}_\gamma$ is a closed Riemann surface of genus $\gamma$ and $p_j \in \bar{M}_\gamma$ ($j = 1, \ldots, k$). ([7])

(ii) The dual Weierstrass data $(G, \omega^2)$ can be extended meromorphically to $\bar{M}_\gamma$. ([2],[27])

We call the points $p_j$ the ends of $f$. An end $p_j$ of $f$ is called regular if the hyperbolic Gauss map $G$ has at most a pole at $p_j$ [23]. By Theorem 1.4, each end of an algebraic CMC-1 surface is regular.

### 2. Ramification estimate for the hyperbolic Gauss map of algebraic CMC-1 surfaces

Next, we give a ramification estimate for the hyperbolic Gauss map of algebraic CMC-1 surfaces.

**Theorem 2.1** ([11]). Let $f: M = \bar{M}_\gamma \setminus \{p_1, \ldots, p_k\} \rightarrow \mathcal{H}^3$ be a non-flat algebraic CMC-1 surface, $G: M \rightarrow \mathbb{C} \cup \{\infty\}$ be the hyperbolic Gauss map of $f$ and $d$ be the degree of $G$ considered as a map $\bar{M}_\gamma$. Assume that, for some fixed distinct values $a_1, \ldots, a_q \in \mathbb{C} \cup \{\infty\}$, $\nu_j$ be the minimum of multiplicity of $G$ at $G^{-1}(a_j)$ (If $a_j$ is the exceptional value of $G$, we put $\nu_j = \infty$). Then we have

$$\sum_{j=1}^q \left(1 - \frac{1}{\nu_j}\right) \leq 2 + \frac{2}{R}, \quad \frac{1}{R} = \frac{\gamma - 1 + k/2}{d} < 1.$$  

(2.1)

As a corollary, we obtain the following result.
Corollary 2.2 ([4], [11]). The hyperbolic Gauss map of non-flat algebraic CMC-1 surface in $\mathcal{H}^3$ can omit at most three values.

However, we do not know the best possible upper bound of the number of exceptional values of $G$ for this class. In fact, we do not find algebraic CMC-1 surfaces whose hyperbolic Gauss map omit 3 values, but there are some algebraic CMC-1 surfaces whose hyperbolic Gauss map omit 2 values (for example, catenoid cousins, examples in [21, Theorem 4.7] and [11, Proposition 2.7]).

The upper bound “$2 + 2/R$” of (2.1) has a geometric meaning. The geometric meaning of “2” is the Euler number of the Riemann sphere, that is, the target space of $G$. Moreover, the geometric meaning of “$R^{-1}$” is as follows: We assume that the universal covering of $M = M_\gamma \setminus \{p_1, \ldots, p_k\}$ is the unit disk $\mathbb{D}$. Let $A_{hyp}(M)$ be the hyperbolic area of $M$ with respect to the hyperbolic metric with Gaussian curvature $-4\pi$ on $\mathbb{D}$ and $A_{FS}$ be the area of $M$ with respect to the induced Fubini-Study metric $G^*\omega_{FS}$ with Gaussian curvature $4\pi$ on the Riemann sphere. Then we have

$$\frac{1}{R} = \frac{\gamma - 1 + k/2}{d} = \frac{A_{hyp}(M)}{A_{FS}(M)}.$$  \hspace{1cm} (2.2)

For more detail on this, see [9, Section 6].

The proof of Theorem 2.1 consists two parts. One is “$R^{-1} < 1$”. This corresponds to the Oseerman type inequality [26] for algebraic CMC-1 surfaces in $\mathcal{H}^3$. The other is the left side of the system of inequalities (2.1). Its proof is based on [9, Theorem 3.3]. For more detail of the proof of Theorem 2.1, see [11].

3. Value distribution of the hyperbolic Gauss maps of flat fronts

Finally, we give an effective estimate for the number of exceptional values of the hyperbolic Gauss maps of flat fronts in $\mathcal{H}^3$. Before that, we briefly recall definitions and basic facts on flat fronts in $\mathcal{H}^3$. For more details, we refer the reader to [6], [13], [14] and [15]. A smooth map $f: M \to \mathcal{H}^3$ from a 2-manifold $M$ is called front if there exists a Legendrian immersion $L_f: M \to T^*_1 \mathcal{H}^3$ into the unit tangent bundle $T^*_1 \mathcal{H}^3$ of $\mathcal{H}^3$ whose projection is $f$, which may have singular points (points $x \in M$ where rank$(df)_x < 2$). Identifying $T^*_1 \mathcal{H}^3$ with the unit normal tangent bundle $T^*_1 \mathcal{H}^3$, $L_f$ corresponds to the unit normal vector field $\nu$ of $f$, that is, the immersion $(f, \nu): M^2 \to T^*_1 \mathcal{H}^3$ satisfies $\langle \nu, df \rangle = 0$. A point which is not singular is said to be regular, where the first fundamental form is positive definite. The Gaussian curvature is well-defined at regular points. A front is said to be flat if the Gaussian curvature vanishes at each regular point (However, this definition of “flat” is not suitable when all points of the front are degenerate. For more details and the other definition of “flat”, see [15, Definition 2.1 and Remark 2.2]).

A front $f$ is said to be complete if there is a symmetric tensor $T$ on $M$ which has compact support such that $T + ds^2$ is a complete Riemannian metric on $M$, where $ds^2$ is the first fundamental form of $f$. For a flat front $f: M \to \mathcal{H}^3$, $M$ is orientable and can be regarded as a Riemann surface such that the second fundamental form is Hermitian. Moreover if $f$ is complete, then there exists a closed Riemann surface $M_\gamma$ of genus $\gamma$ such that $M$ is biholomorphic to $M_\gamma \setminus \{p_1, \ldots, p_k\}$. The points $p_1, \ldots, p_k$ are called the ends of $f$. 


For each point \( p \in M \), there exists a pair \((G(p), G_*(p)) \in S^2_\infty \times S^2_\infty \) of distinct points on the ideal boundary \( S^2_\infty \) such that the geodesic in \( H^3 \) starting from \( G_*(p) \) towards \( G(p) \) coincides with the oriented normal geodesic at \( p \). The pair of the maps

\[
G, G_* : M \to S^2_\infty
\]

are called the hyperbolic Gauss maps of \( f \). If a front \( f \) is flat and we regard \( S^2_\infty \) as the Riemann sphere, then both \( G \) and \( G_* \) are holomorphic. An end \( p_j \) of a flat front \( f \) is said to be regular, if both \( G \) and \( G_* \) can be extended meromorphically across it. Kokubu, Umehara and Yamada showed the following global property for this class.

**Theorem 3.1** ([15], Theorem 3.13). Let \( f : M_\gamma \setminus \{p_1, \ldots, p_k\} \to H^3 \) be a complete flat front whose ends are all regular. Then

\[
d + d_* \geq k
\]

where \( d \) is the degree of \( G \) considered as maps \( M_\gamma \) (if \( G \) has essential singularities, then we define \( \deg G = \infty \)) and \( d_* \) is the degree of \( G_* \) considered as the same way. Furthermore, equality holds if and only if all ends are embedded.

As an application of the inequality, we show the following effective estimate for the number of exceptional values of the hyperbolic Gauss maps for this class.

**Theorem 3.2** ([12]). Let \( f : M_\gamma \setminus \{p_1, \ldots, p_k\} \to H^3 \) be a complete flat front and the maps \( G \) and \( G_* \) be the hyperbolic Gauss maps of \( f \). If \( G \) omits \( p \) values and \( G_* \) omits \( q \) values, then \( p \leq 2 \) or \( q \leq 2 \) or

\[
\frac{1}{p - 2} + \frac{1}{q - 2} \geq \frac{k}{2\gamma - 2 + k}.
\]

Note that the right side of the inequality has no data of the hyperbolic Gauss maps and only topological invariants of \( M = M_\gamma \setminus \{p_1, \ldots, p_k\} \). As a corollary, we give more sharp results on the number of exceptional values of them for some topological cases.

**Corollary 3.3** ([12]). For flat fronts in \( H^3 \), we have the following:

(i) If \( \gamma = 0 \), \( p \geq 4 \) and \( q \geq 4 \), then there does not exist such a front.

(ii) If \( \gamma = 1 \), \( p \geq 5 \) and \( q \geq 5 \), then there does not exist such a front.

**References**


Graduate School of Mathematics, Kyushu University, 6-10-1, Hakoizaki, Higashiku, Fukuoka-city, 812-8581, JAPAN
E-mail address: kawakami@math.kyushu-u.ac.jp
List of MI Preprint Series, Kyushu University
The Global COE Program
Math-for-Industry Education & Research Hub

MI

MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Abstract collision systems simulated by cellular automata

MI2008-2 Eiji ONODERA
The initial value problem for a third-order dispersive flow into compact almost Hermitian manifolds

MI2008-3 Hiroaki KIDO
On isosceles sets in the 4-dimensional Euclidean space

MI2008-4 Hirofumi NOTSU
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme

MI2008-5 Yoshiyasu OZEKI
Torsion points of abelian varieties with values in infinite extensions over a p-adic field

MI2008-6 Yoshiyuki TOMIYAMA
Lifting Galois representations over arbitrary number fields

MI2008-7 Takehiro HIROTsu & Setsuo TANIGUCHI
The random walk model revisited

MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition

MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA
Alpha-determinant cyclic modules and Jacobi polynomials
MI2008-10 Sangyeol LEE & Hiroki MASUDA
Jarque-Bera Normality Test for the Driving Lévy Process of a Discretely Observed Univariate SDE

MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA
A third order dispersive flow for closed curves into almost Hermitian manifolds

MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO
On the $L^2$ a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator

MI2008-13 Jacques FARAUT and Masato WAKAYAMA
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials

MI2008-14 Takashi NAKAMURA
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality

MI2008-15 Takashi NAKAMURA
Some topics related to Hurwitz-Lerch zeta functions

MI2009-1 Yasuhide FUKUMOTO
Global time evolution of viscous vortex rings

MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI
Regularized functional regression modeling for functional response and predictors

MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI
Variable selection for functional regression model via the $L_1$ regularization

MI2009-4 Shuichi KAWANO & Sadanori KONISHI
Nonlinear logistic discrimination via regularized Gaussian basis expansions

MI2009-5 Toshiro HIRANOUCHI & Yuichiro TAGUCHII
Flat modules and Groebner bases over truncated discrete valuation rings
MI2009-6  Kenji KAJIWARA & Yasuhiro OHTA
Bilinearization and Casorati determinant solutions to non-autonomous 1+1
dimensional discrete soliton equations

MI2009-7  Yoshiyuki KAGEI
Asymptotic behavior of solutions of the compressible Navier-Stokes equation
around the plane Couette flow

MI2009-8  Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI
Nonlinear regression modeling via the lasso-type regularization

MI2009-9  Takeshi TAKAISHI & Masato KIMURA
Phase field model for mode III crack growth in two dimensional elasticity

MI2009-10 Shingo SAITO
Generalisation of Mack’s formula for claims reserving with arbitrary exponents
for the variance assumption

MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA
Ultradiscretization of a solvable two-dimensional chaotic map associated with
the Hesse cubic curve

MI2009-12 Tetsu MASUDA
Hypergeometric Ψ-functions of the q-Painlevé system of type $E_8^{(1)}$

MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic De-
composition for Quantifier Elimination

MI2009-14 Yasunori MAEKAWA
On Gaussian decay estimates of solutions to some linear elliptic equations and
its applications

MI2009-15 Yuya ISHIHARA & Yoshiyuki KAGEI
Large time behavior of the semigroup on $L^p$ spaces associated with the lin-
earized compressible Navier-Stokes equation in a cylindrical domain
MI2009-16 Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE
Spectrum in multi-species asymmetric simple exclusion process on a ring

MI2009-17 Masato WAKAYAMA & Keitaro YAMAMOTO
Non-linear algebraic differential equations satisfied by certain family of elliptic functions

MI2009-18 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of an Elliptical Flow Subjected to a Coriolis Force

MI2009-19 Mitsunori KAYANO & Sadanori KONISHI
Sparse functional principal component analysis via regularized basis expansions and its application

MI2009-20 Shuichi KAWANO & Sadanori KONISHI
Semi-supervised logistic discrimination via regularized Gaussian basis expansions

MI2009-21 Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations

MI2009-22 Eiji ONODERA
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces

MI2009-23 Stjepan LUGOMER & Yasuhide FUKUMOTO
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions

MI2009-24 Yu KAWAKAMI
Recent progress in value distribution of the hyperbolic Gauss map